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# Separability, Partial Demand Systems, and Consumer's Surplus Measures[1]

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In practice, *complete* demand systems are not estimated. Rather, either an *incomplete* demand system is estimated, or separability is invoked and a *partial* demand system is estimated. This paper considers the relationship between the conventional compensating variation (equivalent variation) and the corresponding welfare measure that can be derived from a *partial* demand system and the current budget allocation to the separable group. Even assuming the separability assumption invoked is appropriate, these *partial* measures provide, in general, only a limited amount of information about the compensating variation and no information about the equivalent variation. Great care is therefore needed when using *partial* welfare measures to evaluate policy. 

## I. INTRODUCTION

Consider the conventional compensating variation, CV, and equivalent variation, EV, associated with a proposed policy that, if enacted, would change the prices of some market commodities and/or change the quantities of some nonmarket commodities. Then assume that these market and nonmarket commodities belong to a group that is separable from all other commodities. Given this separability, a system of *partial* demand functions exists for the market commodities in this group such that demand is just a function of the prices of these market commodities, the budget allocation to this group, and the quantities of the separable nonmarket commodities. From this system of *partial* demand functions, one can derive the partial CV and EV for any proposed change in the group's exogenous prices and quantities as a function of the budget allocation to the group. These *partial* welfare measures can be evaluated at the current budget allocation to the group or, if known, what would be the group's optimal budget in the proposed state. We develop, and attempt to motivate, the theory that identifies the relationship between the conventional CV (EV) associated with a proposed change in the prices and/or quantities in this separable group, and these *partial* welfare measures.

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The conventional CV and EV are both exact welfare measures (Morey [8]). For example, if $CV_i > 0$ ($CV_i < 0$) one knows the proposed policy will make individual $i$ better off (worse off). In addition, $\Sigma CV_i > 0$ is a necessary condition for a policy to be a potential Pareto improvement. The issue in much applied welfare analysis is how to estimate the CV and EV for some change in a subset of market prices and/or quantities of nonmarket commodities. As is well known, an individual's CV or EV can be easily derived if one knows the individual's indirect utility function or expenditure function, and that knowledge of the individual's *complete* demand system is equivalent to a knowledge of either of these functions. In theory, one can therefore estimate an individual's CV or EV by first estimating their complete demand system. However, in practice, it is rarely, if ever, possible to estimate a complete demand system. Complete price and consumption data are just not available. What happens in practice is the analyst, interested in only a subset of commodity space, estimates an *incomplete* demand system or a *partial* demand system. An *incomplete* demand system is a subset of the demand functions in a *complete* demand system. For the welfare analyst, whether it is better to estimate an *incomplete* or a *partial* demand system depends on the available data, whether the policy to be evaluated involves a change in the level of nonmarket commodities, and whether the analyst is more comfortable with a separability assumption or an assumption that the unobserved variables in the *incomplete* demand system do not vary across the sample. The problem for welfare analysis is that neither an *incomplete* demand system nor a *partial demand* system contain sufficient information about preferences to completely derive the underlying indirect utility function. This raises two important policy questions: (i), What can one determine about the CV or EV for a given change in prices and/or nonmarket commodities from a *incomplete* demand system? and (ii), What can one determine about the CV or EV for a given change in prices and/or nonmarket commodities from a *partial* demand system? The concern of this paper is this second question. The derivation of welfare measures from *incomplete* demand systems is considered, in detail, by Hausman [4] and LaFrance and Hanemann [7]. Briefly, and without qualification, they show that from an *incomplete* demand system one can derive the CV (or EV) associated with a change in the *prices* of the commodities included in the incomplete system, but that from an *incomplete* demand system one cannot, in general, derive the CV (or EV) associated with a change in the quantities of the nonmarket commodities. *Incomplete* demand systems, and the derivation of welfare measures from them, are discussed in more detail in Footnote 3.

Returning to the practical significance of the *partial* CV and EV, consider cases where the analyst chooses to assume separability, estimates *partial* demand system, and derives a partial CV (EV). Consider those cases where the separability assumption is appropriate.[2] The policy relevance of these estimated *partial* measures depends on whether they can be used to determine whether a proposed policy will make an individual better or worse off, and whether necessary conditions are fulfilled for a potential pareto improvement. This paper answers these questions.

[2]If the analyst is mistaken and the assumption of separability is erroneous, the "supposed" *partial* welfare measures provide, in general, no information about the CV or EV. Our main concern is not with the consequences when separability is erroneous, but the consequences when separability holds and the analyst estimates a *partial* demand system. The consequences of erroneously assuming separability are briefly discussed in Section VI.

A major, but not exclusive, reason for invoking separability is lack of data. For example, consider a situation where the analyst is only concerned with estimating the consumer's surplus that would result from a proposed change in the price and/or attributes of a small group of commodities, such as recreational sites or food stuffs. Assume the analyst has data on the prices, attributes, and consumption levels for those commodities, but does not have data on the prices or attributes of other commodities. Assuming there is variation in these unobserved prices or attributes, one cannot estimate an *incomplete* demand system for the commodities of interest. However, one can either explicitly or implicitly assume that the commodities under consideration, and their attributes, are separable from all other commodities. For example, site-specific recreational activities are often assumed separable from all the other commodities in the consumer's choice set. Assuming separability is appropriate, one can estimate the system of *partial* demand functions for these commodities with just data on the prices of these commodities, the attributes of these commodities, and the budget allocation to the group. This approach provides either an explicit or implicit estimate of the *partial* expenditure function for this group of commodities. This *partial* expenditure function, along with the current budget allocation to this group, can then be used to estimate a *partial* CV and EV associated with any proposed change in the attributes of these commodities. This approach has been taken in many recreational demand studies, including some by one of the authors; see, for example, Morey [9]. However, even accepting the separability assumption, whether this "shortcut" approach to consumer's surplus estimation is of value depends completely on the relationship between these *partial* measures and the conventional CV, and EV. However, this relationship is not well understood. Morey [9], for example, states that "the magnitude of the (estimated) compensating variation does not depend on the fact that skiing activities are weakly separable from all other activities." He also states that his estimated equivalent variation is a lower bound estimate on the true equivalent variation. On both counts, he is incorrect.

Another example of *partial* measures arises when nonmarket commodities such as environmental quality and the goods provided by the different levels of the government are not included as dependent variables in the representative individual's demand functions for market commodities. Assuming that there is variation in these nonmarket commodities, this omission cannot be justified without invoking the assumption that the market commodities for which demand is being estimated are separable from these nonmarket commodities. If this justification is adopted, then one must be cognizant of the fact that he or she is dealing with a *partial* demand system and that any consumer's surplus measures derived from this system are just *partial* measures. An analogous point could be made for other commodities such as leisure and financial assets, which likely influence demand but do not explicitly appear in the demand function.

As a final example of *partial* measures, consider a situation where the policy analyst desires the consumer's surplus that would result from a proposed change in the prices of a group of current-period market commodities. Assume, not unrealistically, that preferences are defined over the consumption of both current and future commodities, but that price and consumption data are only available for current-period commodities. Assuming current commodities are separable in the utility function, the CV obtained by specifying and estimating a current period expenditure function is only a *partial* CV.

Despite the need for a theory relating the *partial* CV (EV) to the conventional CV (EV), there has been little recognition that such a theory is required. We have been able to find only two articles on this topic, both of which restrict themselves to considering the issue in the context of price changes and intertemporal utility. Blackorby, Donaldson, and Moloney [1] consider *partial* and conventional CVs (EVs) in the context of an intertemporal utility function that is separable by periods and where the optimal budget allocation to each period, *in the proposed state*, is known. They consider a change in the vector of spot prices and show that the discounted sum of each period's *partial* CV, each evaluated at the period's budget allocation that will be optimal in the proposed state, is not an exact welfare measure. Expanding on their results, Keen [6] investigates the degree of bias in the sum of the per-period *partial* CVs, but only for price changes and under the restrictive assumption that the utility function is *additively* separable across periods.

We consider *partial* measures evaluated at both the current budget allocation and the budget allocation that would be optimal in the proposed state but emphasize the *partial* CV and EV evaluated at the group's current budget allocation. We feel that these are the most relevant *partial* measures because these are the *partial* measures that can be estimated with just existing data on the separable group. The current budget allocation to the separable group can be observed, but we cannot currently observe what the budget allocation will be in the proposed state. In addition, a group's optimal budget allocation in the proposed state cannot be predicted from just the group's *partial* demand system.

The paper is organized as follows. For a proposed change in the prices of a group of market commodities and/or a proposed change in the quantities of a group of nonmarket commodities, three distinct sets of compensating and equivalent welfare measures are defined. Section II defines the conventional compensating variation (CV) and equivalent variation (EV). Both measures are derived from the full expenditure function. Section III defines the *partial* CV and the *partial* EV. These are the measures that can be derived from the *partial* expenditure function for the group of commodities under study. Section IV defines a *constrained* CV and a *constrained* EV. These are the welfare measures when the individual is constrained to consume the same vector of other market commodities after the prices and quantities change in the group under study. While the individual is usually not constrained in this way, consideration of these *constrained* measures will enhance our understanding of the *partial* measures and their link with the conventional measures. Section V outlines the theoretical relationships between the *partial*, the *constrained*, and the conventional CV and EV measures. Section VI summarizes and discusses the policy implications of our results.

## II. THE COMPENSATING AND THE EQUIVALENT VARIATION

Define market commodities as the goods and services that can be purchased by the individual at parametric prices. These are the individual's choice variables. Define all the other factors that affect utility but are exogenous to the individual as nonmarket commodities; these include environmental quality, public goods, and the attributes (characteristics) of the market commodities.

Assume that the individual's preferences for commodities can be represented by the direct utility function

$$u = u(x, b, z, c), \tag{1}$$

where $x \equiv (x_1, x_2, \ldots, x_N)'$ is the subset of market commodities for which there are price and consumption data; $z \equiv (z_1, z_2, \ldots, z_M)'$ is the set of market commodities for which there are no data; $b \equiv (b_1, b_2, \ldots, b_K)'$ is the set of nonmarket commodities for which there are consumption data; and $c \equiv (c_1, c_2, \ldots, c_L)'$ is the subset of nonmarket commodities for which there are no data.

This preference ordering can also be represented by the indirect utility function,

$$u = v(p, b, q, c, y), \tag{2}$$

or the expenditure function,

$$m(p, b, q, c, u), \tag{3}$$

where $y$ is the individual's total income, and $p = (p_1, p_2, \ldots, p_N)'$ and $q = (q_1, q_2, \ldots, q_M)'$ are the commodity price vectors.

Both the indirect utility function, $v(p, b, q, c, y)$, and the expenditure function, $m(p, b, q, c, u)$, can be used to define monetary measures of the welfare effects of a proposed change in the constraints. Assume that the individual currently faces the parametric prices and nonmarket commodities $p^o$, $b^o$, $q^o$, and $c^o$. Given $y^o$, these constraints allow the individual to achieve some maximum utility level $u^o$ by choosing $\{x^o, z^o\}$ which, given prices, implies some total expenditures on the $x$ and $z$ commodities, denoted $y_x^o$ and $y_z^o$, satisfying $y_x^o + y_z^o = y^o$. Now suppose a policy is proposed that, if enacted, would cause the prices of the observed market commodities and the quantities of the observed nonmarket commodities to change to $p^\iota$ and $b^\iota$, with $y$, $q$, and $c$ remaining constant at, respectively, $y^o$, $q^o$, and $c^o$. If this proposal were enacted, the individual would achieve some maximum utility level $u^\iota$ by choosing $\{x^\iota, z^\iota\}$ which, given prices, implies some different allocation of $y^o$, denoted $x_x^\iota$ and $y_z^\iota$. The compensating variation, CV, associated with this proposed change is

$$v(p^\iota, b^\iota, q^o, c^o, y^o - \mathrm{CV}) = v(p^o, b^o, q^o, c^o, y^o) \equiv u^o. \tag{4}$$

The CV is the amount of unrestricted money that the individual would have to pay out (receive) in the proposed state, $\{p^\iota, b^\iota\}$, to make him indifferent between the proposed state with the payment (or receipt) and the current state, $\{p^o, b^o\}$. If $\{p^\iota, b^\iota\}$ is preferred to $\{p^o, b^o\}$ then CV > 0.

The equivalent variation, EV, associated with this same change is

$$u^\iota \equiv v(p^\iota, b^\iota, q^o, c^o, y^o) = v(p^o, b^o, q^o, c^o, y^o + \mathrm{EV}). \tag{5}$$

The EV is the amount of unrestricted money that the individual would have to receive (or pay) in the current state, $\{p^o, b^o\}$, to make him indifferent between the current state with the receipt (or payment) and the proposed state, $\{p^\iota, b^\iota\}$. If $\{p^\iota, b^\iota\}$ is preferred to $\{p^o, b^o\}$ then EV > 0.

In terms of the expenditure function,

$$CV = y^o - m(p^\iota, b^\iota, q^o, c^o, u^o) \tag{6}$$

and

$$EV = m(p^o, b^o, q^o, c^o, u^\iota) - y^o. \tag{7}$$

Information on the *complete* demand system,

$$x_j = h_j(p, b, q, c, y), \qquad j = 1, 2, \ldots, N, \tag{8}$$

$$z_i = f_i(p, b, q, c, y), \qquad i = 1, 2, \ldots, M, \tag{9}$$

is, in general, necessary in order to calculate the CV or EV for any proposed change involving a change in $b$ or $c$, and sufficient, but not necessary, to calculate the CV or EV for any proposed change in $p$ or $q$.[3]

The information in this *complete* demand system is employed to derive the CV (or EV) in one of two ways. One procedure is to start by specifying a direct or indirect utility function and derive from it the formulas for the demand functions; the coefficients of the utility functions can be recovered from those of the fitted demand functions, and these can be used to compute CV or EV from (4) and (5). Alternatively, one starts by specifying and estimating some *complete* demand system, (8) and (9), and then recovers the income compensation function by integrating the differential equations associated with Shephard's Lemma—either analytically, as in Willig [12] and Hausman [4], or numerically, as in Vartia [11]. However, in the absence of data on the current levels of $q$, $c$, and $z$ ($q^o$, $c^o$, and $z^o$, respectively), the *complete* demand system cannot be estimated. Therefore, neither of these procedures for computing the CV or EV can be employed, and an alternative tack must be taken. The common response to a lack of data on $z$, $q$, and $c$ is to either assume $x$ and $b$ form a separable group and estimate a *partial* system for the $x$ commodities, or, alternatively estimate just the $x$ demand functions (an *incomplete* system) assuming $q$ and $c$ do not vary across the sample. Consider the implications of the separability approach.

[3]Note that Eq. (8), by itself, is an *incomplete* demand system which can be estimated with just data on $x$, $p$, $b$, $q$, $c$, and $y$. Estimation of this *incomplete* system does not require data on $z$, but does require data on $q$ and $c$, unless one is willing to assume that $q$ and $c$ do not vary across the sample or do not enter the demand functions for the $x$ commodities. LaFrance and Hanemann [7] prove that the CV and EV for a change in $p$ can be derived from this *incomplete* demand system. Estimation of the *incomplete* demand system, Eq. (8), is therefore an attractive option if one has data on $q$ and $c$, and if one's sole intent is to derive the CV or EV for a change in a *subset of the p prices*.

However, they also demonstrate that the CV (or EV) associated with a change in $b$, or $c$, cannot, in general, be derived from any incomplete demand system. Therefore, estimating an *incomplete* demand system is not, in general, an option if one's intent is to derive the CV or EV associated with a change in nonmarket commodities. For more details, see LaFrance and Hanemann [7].

Finally, as noted above, if one only has data on $x$, $p$, $b$, and $y$, it is still possible to estimate the *incomplete* demand system, Eq. (8), if one is willing to assume that $c$ and $q$ do not vary across the sample. In this case, the given values of $q$ and $c$ are subsumed in the coefficients on $p$, $b$, and $y$. However, if one adopts this assumption, the derived CV and EV for a change in $p$ are only correct for the given, and unknown, $q$ and $c$. If $q$ or $c$ change, they are no longer valid. If the assumption that $q$ and $c$ do not vary is erroneous, the derived CV and EV will have a bias of indeterminate direction and magnitude.

## III. SEPARABILITY AND PARTIAL DEMAND SYSTEMS: A PARTIAL COMPENSATING VARIATION, $\mathrm{CV}_x$, AND A *PARTIAL* EQUIVALENT VARIATION, $\mathrm{EV}_x$

Assume the "data" commodities are separable from all the other commodities in the utility function; i.e.,[4]

$$u = u(x, b, z, c) = \phi[u_d(x, b), z, c], \tag{10}$$

where

$$u_d = u_d(x, b). \tag{11}$$

The function $u_d(x, b)$ is the aggregator function for the "data", $d$, commodities; i.e., $u_d$ is a quality-adjusted quantity index for the $x$ commodities. Call $u_d(x, b)$ the *partial* direct utility function and call $u_d$, "$d$-utility". Following Pollak [10], the individual's system of *partial* demand functions for the $x$ commodities can be obtained as the solution to the constrained optimization problem

$$\max_x u_d(x, b) \qquad \text{subject to } \sum p_j x_j = y_x. \tag{12}$$

These demand functions,

$$x_j = h_{xj}(p, b, y_x), \qquad j = 1, 2, \ldots, N, \tag{13}$$

are *partial* in that they are conditional on the budget allocation to the $x$ commodities, $y_x$. Substituting this system of *partial* demand functions into the *partial* direct utility function for the "data" commodities, $u_d(x, b)$, one obtains the *partial* indirect utility function

$$u_d = v_d(p, b, y_x), \tag{14}$$

which may be inverted to obtain the *partial* expenditure function

$$m_d(p, b, u_d). \tag{15}$$

The *partial* expenditure function is expressed in terms of expenditure on $x$, i.e., money that must be spent on the $x$ commodities and is paid out of the $x$ budget, $y_x$.

Assuming the separability assumption is appropriate, one can define the class of *partial* compensating variations, $\mathrm{CV}_x(y_x)$, that the individual would associate with a proposed change in the prices and quantities of the "data" commodities from $\{p^o, b^o\}$ to $\{p^\iota, b^\iota\}$ as

$$v_d(p^\iota, b^\iota, y_x - \mathrm{CV}_x(y_x)) = v_d(p^o, b^o, y_x^o) \equiv u_d^o. \tag{16}$$

[4]Note that this nonsymmetric form of separability is weaker than what is commonly referred to as weak separability. Weak separability exists if $u = u(x, b, z, c) = \Psi[u_d(x, b), u_n(z, c)]$. Blackorby, Primont, and Russell [2] classify the separability implied by (10) as a type of recursive separability.

The two $\mathrm{CV}_x(y_x)$ that are of most relevance are $\mathrm{CV}_x(y_x^o)$ and $\mathrm{CV}_x(y_x^\iota)$, i.e., the *partial* CV evaluated at the optimal budget allocation to the $x$ group in the current state, $y_x^o$, and the *partial* CV evaluated at what would be the optimal allocation to the $x$ group in the proposed state, $y_x^\iota$. Note that

$$\mathrm{CV}_x(y_x^\iota) = \mathrm{CV}_x(y_x^o) + \left(y_x^\iota - y_x^0\right), \tag{17}$$

$\mathrm{CV}_x(y_x^\iota)$ is an $x$-restricted money metric of the $d$-utility change caused by the change from $\{p^o, b^o\}$ to $\{p^\iota, b^\iota\}$, i.e.,

$$\mathrm{CV}_x(y_x^\iota) \geq 0 \Leftrightarrow u_d^\iota \geq u_d^o, \tag{18}$$

where

$$u_d^\iota \equiv v_d(p^\iota, b^\iota, y_x^\iota). \tag{19}$$

However, whereas $y_x^o$ is observed, the budget allocation to the $x$ commodities in the proposed state, $y_x^\iota$, is not observed and cannot be predicted from the *partial* demand functions (13). $\mathrm{CV}_x(y_x^o)$ is therefore the *partial* measure that is most often estimated. Because of this, we concentrate on $\mathrm{CV}_x(y_x^o)$ and its relationship with the CV. For brevity let $\mathrm{CV}_x$ denote $\mathrm{CV}_x(y_x^o)$. In terms of the *partial* expenditure function,

$$\mathrm{CV}_x = y_x^o - m_d(p^\iota, b^\iota, u_d^o), \tag{20}$$

$\mathrm{CV}_x$ is the amount the individual's expenditures on $x$ would have to increase or decrease in the proposed state, $\{p^\iota, b^\iota\}$, to make the maximum $d$-utility in the proposed state equal to $u_d^o$. Given a sample of individuals that contains data on $y_x^o$, $p^o$, $x^o$, and $b^o$, the *partial* demand system (13) can be estimated and, from this, one can recover the $d$-utility function. Using either of the methods mentioned above, the information on the *partial* demand system can be used to compute $\mathrm{CV}_x$. However, $\mathrm{CV}_x$ does not, in general, equal CV. Assuming separability, $\mathrm{CV}_x$ can be estimated given a sample that does not contain data on $q^o$, $z^o$, or $c^o$.

Similarly, define the class of *partial* equivalent variations, $\mathrm{EV}_x(y_x)$, that the individual would associate with a proposed change from $(p^o, b^o)$ to $(p^\iota, b^\iota)$ as

$$u_d^\iota(y_x) \equiv v_d(p^\iota, b^\iota, y_x) = v_d\left(p^o, b^o, y_x^o + \mathrm{EV}_x(y_x)\right). \tag{21}$$

Two members of this class are $\mathrm{EV}(y_x^o)$ and $\mathrm{EV}(y_x^\iota)$:

$$\mathrm{EV}_x(y_x^\iota) = \mathrm{EV}_x(y_x^o) + (y_x^\iota - y_x^o). \tag{22}$$

Remembering that $y_x^o$ is observed but that $y_x^\iota$ is not observed and cannot be predicted from the partial demand functions (13), we concentrate on $\mathrm{EV}_x(y_x^o)$, and for brevity let $\mathrm{EV}_x$ denote $\mathrm{EV}_x(y_x^o)$. In terms of the *partial* expenditure function for the $x$-commodities

$$\mathrm{EV}_x = m_d\left(p^o, b^o, u_d^\iota(y_x^o)\right) - y_x^o, \tag{23}$$

where

$$u_d^\iota(y_x^o) \equiv v_d(p^\iota, b^\iota, y_x^o). \tag{24}$$

$\mathrm{EV}_x$ is the amount of the individual's expenditures on $x$ in the current state would have to increase or decrease to make the maximum $d$-utility in the current state equal to $u_d^{\iota}(y_x^o)$. For the proposed change, $\mathrm{EV}_x$ can be estimated assuming separability and given a sample that only contains data on $y_x^o$, $p^o$, $x^o$, and $b^o$.

## IV. THE CONDITIONAL CONSUMER'S SURPLUS MEASURES

One's understanding of the *partial* welfare measures is enhanced by considering a situation where the individual is assumed constrained to continue to consume $z^o$ after prices and attributes have changed from $\{p^o, b^o\}$ to $\{p^{\iota}, b^{\iota}\}$.[5] Though the individual is not, in fact, constrained in this way, it is insightful to define a *constrained* CV and a *constrained* EV in this context. We will show, in Section V, that this *constrained* CV equals $\mathrm{CV}_x$ and this *constrained* EV and $\mathrm{EV}_x$. These equivalences provide more intuitive interpretations of $\mathrm{CV}_x$ and $\mathrm{EV}_x$ and a criterion by which to determine when knowledge of these *partial* measures is useful for policy evaluation.

Define

$$\hat{v}(p, b, q, c, y, z) \tag{25}$$

as the *conditional* indirect utility function when the individual is constrained to consume $z$. Corresponding to this *conditional* indirect utility function is the *conditional* expenditure function

$$\hat{m}(p, b, q, c, u, z). \tag{26}$$

Given a sample of individuals that contains data on $y^o, x^o, b^o, p^o, q^o, c^o$, and the constrained amount $z$, one can recover the conditional expenditure function, (26), and the *conditional* indirect utility function, (25), from the system of *conditional* demand functions[6]

$$x_j = \hat{h}_j(p, b, q, c, y, z), \qquad j = 1, 2, \ldots, N. \tag{27}$$

The functions $\hat{v}(p, b, q, c, y, z)$ and $\hat{m}(p, b, q, c, u, z)$ can be used to define a *constrained* CV, $\hat{\mathrm{CV}}$, and a *constrained* EV, $\hat{\mathrm{EV}}$, associated with a proposed change from $\{p^o, b^o\}$ to $\{p^{\iota}, b^{\iota}, z^o\}$.

*Define* $\hat{\mathrm{CV}}$.

$$\begin{aligned} v(p^o, b^o, q^o, c^o, y^o) &= \hat{v}(p^o, b^o, q^o, c^o, y^o, z^o) \\ &= \hat{v}\left(p^{\iota}, b^{\iota}, q^o, c^o, y^o - \hat{\mathrm{CV}}, z^o\right). \end{aligned} \tag{28}$$

$\hat{\mathrm{CV}}$ is the amount the individual's expenditures on $x$ would have to increase or decrease in the proposed state with the constraint, $\{p^{\iota}, b^{\iota}, z^o\}$, to make him

[5]Note that constraining the individual to consume $z^o$ in the new state, where $z^o$ was the optimal $z$ in the original state, is not equivalent to making $z$ a nonmarket commodity. A nonmarket commodity is constrained in both states.

[6]The relationship between the *conditional* demand function, Eq. (27), and the unconditional demand function, Eq. (8), is developed in Neary and Roberts [5] and Cornes and Albon [3].

indifferent between the proposed constrained state and the current unconstrained state, $\{p^o, b^o\}$.

*Define* $\mathrm{E\hat{V}}$.

$$\hat{u}^\iota \equiv \hat{v}(p^\iota, b^\iota, q^o, c^o, y^o, z^o) = \hat{v}(p^o, b^o, q^o, c^o, y^o + \mathrm{E\hat{V}}, z^o). \tag{29}$$

$\mathrm{E\hat{V}}$ is the amount the individual's expenditures on $x$ would have to increase or decrease in the current unconstrained state, $\{p^o, b^o\}$, to make him indifferent between the current state and the proposed state with the constraint, $\{p^\iota, b^\iota, z^o\}$.

In terms of the *conditional* expenditure function

$$\mathrm{C\hat{V}} = y^o - \hat{m}(p^\iota, b^\iota, q^o, c^o, u^o, z^o) \tag{30}$$

and

$$\mathrm{E\hat{V}} = \hat{m}(p^o, b^o, q^o, c^o, \hat{u}^\iota, z^o) - y^o. \tag{31}$$

## V. THE RELATIONSHIPS BETWEEN THE DIFFERENT COMPENSATING AND EQUIVALENT VARIATIONS

### *A. The Relationships between the Different Compensating Variations*

LEMMA 1.

$$\textit{Given separability,} \qquad \mathrm{CV}_x = \mathrm{C\hat{V}}. \tag{32}$$

The proof is in the appendix.

Lemma 1 implies that $\mathrm{CV}_x$ does not have to be expressed in terms of $d$-utility, but rather can be defined as the amount the individual's expenditures on $x$ would have to increase or decrease in the proposed state with the constraint, $\{p^\iota, b^\iota, z^o\}$, to make him indifferent between the proposed constrained state and the current unconstrained state, $\{p^o, b^o\}$. Lemma 1 also shows that $\mathrm{CV}_x$ is the desired compensating variation only if the individual is constrained to consume the same vector of the other market commodities, $z^o$, after the prices of the $x$ commodities and/or the quantities of the $b$ commodities change. The individual is *not usually* constrained in this way.

THEOREM 1.

$$\textit{Given separability,} \qquad \mathrm{CV}_x \leq \mathrm{CV}. \tag{33}$$

*Proof.* Substituting the definitions of CV and $\mathrm{C\hat{V}}$ (Eqs. (6) and (30), respectively) into $(\mathrm{CV} - \mathrm{C\hat{V}})$, one obtains $[\hat{m}(p^\iota, b^\iota, q^o, c^o, u^o, z^o) - m(p^\iota, b^\iota, q^o, c^o, u^o)]$. This amount is nonnegative because adding an additional constraint cannot lower minimum costs. This proves that $\mathrm{C\hat{V}} \leq \mathrm{CV}$. Given $\mathrm{C\hat{V}} \leq \mathrm{CV}$, and $\mathrm{CV}_x = \mathrm{C\hat{V}}$ (Lemma 1), $\mathrm{CV}_x \leq \mathrm{CV}$. Q.E.D.

Intuitively, $\mathrm{CV}_x$ provides a lower bound on the CV because improvements and/or deteriorations will often cause the individual to change his budget allocation between the $x$ and $z$ commodities. The CV, but not the $CV_x$, incorporates this budget adjustment that the individual will choose to make. An individual will pay less to bring about an improvement, $\{p^o, b^o\}$ to $\{p^\iota, b^\iota\}$, if he is constrained in his ability to take advantage of that improvement. Holding the budget allocation between the $x$ and $z$ commodities at their original levels ($y_x^0$ and $y_z^o$) is one such constraint. Similarly, if $\{p^o, b^o\}$ to $\{p^\iota, b^\iota\}$ is a deterioration, the individual will have to be paid more to accept this proposed state when he cannot reduce the impact of the deterioration by reallocating his budget between the $x$ and $z$ commodities.

An immediate corollary of Theorem 1 is that if $z$ is empty (i.e., if *all* market commodities are included in the *partial* utility function), then $CV_x = CV$. This is because when the $x$ vector includes all market commodities there is no distinction between expenditures and expenditures on $x$. In contrast, when market data are not complete (i.e., $z$ has positive dimension), $CV_x$ will not, in general, equal the CV even if the partial utility function includes *all* of the nonmarket commodities (i.e., even if $c$ is empty).

Four other policy relevant corollaries also follow from Theorem 1: (i) $CV_x > 0$ is a sufficient, but not a necessary, condition for $CV > 0$; (ii) $CV_x < 0$ tells one nothing about the sign of the CV; (iii) $\Sigma_i CV_{xi} > 0$ is a sufficient, but not necessary, condition to fulfill a necessary condition for the policy to be a potential Pareto improvement; and (iv) $\Sigma_i CV_{xi} < 0$ tells one nothing about whether the policy is a potential Pareto improvement.

Given that $CV_x$ is only a lower bound on the CV, it is of interest to briefly consider how closely $CV_x$ bounds the CV.[7] Put simply, it depends on the preferences of the individual, and the bound can be close or far. Consider first sufficient assumptions to imply that $CV_x = CV$. Assume the *partial* demand functions, Eq. (13), exhibit zero income effects for some subset of the $x$ commodities.[8] In such a case, it can be shown that each partial demand function that exhibits zero income effects coincides with its ordinary demand functions, Eq. (8), and its Hicksian demand function. This follows because the ordinary and Hicksian demand function coincide when there are zero income effects, and if a *partial* demand function has zero income effects this implies zero income effects for the corresponding demand function. Accordingly, as long as the price changes are confined to the $x$ commodities with zero income effects, and the changes in the levels of the nonmarket commodities, $b$, are confined to those elements of $b$ that are weakly complementary with those $x$ commodities, it will be the case that $CV_x = CV$, and, more generally, that $CV_x = CV = EV = EV_x$. Note, that since zero income effects for most commodities is unlikely, one cannot expect this equality to hold.

More generally, when income effects are present in the affected commodities, $|CV - CV_x|$ depends, roughly speaking, on the marginal rate of substitution between the separable group (the $x$ and $b$ commodities) and the "group" of all other

[7]While discussed here in general terms, a rigorous analysis of this issue is beyond the scope of this paper. For an introduction to the formalities of determining the degree of bias in $CV_x(y_x^o)$ see Keen [6] who, in an intertemporal context without nonmarket commodities, considers the degree of bias when the utility function is restrictively assumed to be additively separable.

[8]Note that, given the utility function, Eq. (10), at most $N - 1$ of the $x$'s can have partial demand functions with zero income effects.

commodities ($z$ and $c$).[9] Note that this marginal rate of substitution *cannot* be determined from a *partial* demand system. *Ceteris paribus*, for a given change in $p$ and/or $b$, the CV decreases, in absolute value, as it becomes easier, in terms of preferences, to substitute in, and out, of the separable group. This is because the importance of what happens in the separable group diminishes as the substitutability between the separable group and other commodities increases. However, as noted, $CV_x$ does not account for how easily the individual substitutes between the $x$ and $z$ commodities. $CV_x$ implicitly holds the budget allocation to the $x$ group, $y_x$, constant when $p$ and/or $b$ changes. This generates the downward bias of $CV_x$. The degree of this bias is an increasing function of how much the individual would have like to adjust $y_x$ in response to the change in $p$ and $b$, and the desired adjustment in $y_x$ is an increasing function of how easy it is, in terms of preferences, to substitute in and out of the $x$ group.

It may be useful to illustrate these points with a numerical example. While a numerical example is, by definition, only a special case, it will provide some impression of the potential degree of bias in the $CV_x$ and how the degree of bias varies as a function of the degree of substitutability between market commodities. Consider an individual with the nested CES preference ordering

$$u = u(x, b, z, c) = \left[\alpha_1 b_1 x_1^{\beta} + \alpha_2 b_2 x_1^{\beta}\right]^{\rho} + \alpha_3 c z^{\rho}, \tag{34}$$

where we know the specific values of $\alpha$, $\beta$, and $\rho$.[10] This simple two-level CES preference ordering was chosen for the numerical example because it restricts the elasticity of substitution between commodities $x_1$ and $x_2$ to $\sigma_0 = 1/(1-\beta)$, and the elasticity of substitution between commodity $z$ and the $x$ aggregate to $\sigma = 1/(1-\rho)$. One can therefore hold the degree of substitutability between $x_1$ and $x_2$ constant and increase the degree of substitutability between $z$ and the $x$ aggregate by just increasing $\sigma$.[11] For the numerical example, assume $\alpha_1 = 1$, $\alpha_2 = 1.4$, $\alpha_3 = 1.5$, and $\beta = 0.5$. Assume that in the initial state $y^o = 100$, $p_1^o = 4$, $p_2^o = 5$, $q^o = 1$, $b_1^o = 1$, $b_2^o = 1$, and $c^o = 1.5$. Table I reports this individual's CV and $CV_x$ for, ceteris paribus, both an increase ($b_1^t = 2.0$) and decrease ($b_1^t = 0.5$) in the quality of $x_1$ as a function of the elasticity of substitution between $z$ and the $x$ aggregate, $\sigma$. Note that the absolute magnitude of the CV and the degree of bias in the $CV_x$ both vary significantly as function of the degree of substitutability between the separable $x$ group and $z$. Further note that, at least in this example, bias in the $CV_x$ can be significant. The less significant the impact of the change, the greater the bias, but, even when the CV in this example is relatively large, the bias is still significant. Too much should not be made of the specific degrees of bias

[9]More formally, note that a quantity index of the ($z$ and $c$) commodities only exists if the $z$ and $c$ commodities form a separable group in $u = u(x, b, z, c)$. As discussed in Footnote 4, this is a stronger separability assumption than was made it Eq. (10) but is required if one wants to formally define the MRS between the quantity index of the ($x$ and $b$) commodities and the quantity index of the ($z$ and $c$) commodities.

[10]Note that only knowledge of $\alpha_1$, $\alpha_2$, and $\beta$ can be obtained from the *partial* demand system for the $x$ commodities.

[11]While attractive for this reason, this preference ordering is highly restrictive, and one should be hesitant to derive too many generalities from it. The constant elasticities of substitution follow from the restriction that the aggregator function, $u_d$, is homothetic and directly additive in $x_1$ and $x_2$, and the restriction that the utility function is directly additive and homothetic in $u_d$ and $z$.

TABLE I

A Numerical Example: An Individual's Compensating Variation, CV, and *Partial* Compensating Variation, $CV_x$, as a Function of the Elasticity of Substitution between the Separable Group and All Other Commodities, $\sigma$

| $\sigma$ | $y_x^o$ | $y_x^t$ | CV | $CV_x$ | $CV_x/CV$ |
|---|---|---|---|---|---|
| | | An improvement in the quality of $x_1$ ($b_1^o = 1.0$, $b_1^t = 2.0$) | | | |
| 10.00 | 0.0005572 | 0.5866 | 0.06529 | 0.0003 | 0.0046 |
| 5.000 | 0.2937 | 6.113 | 1.492 | 0.1583 | 0.1061 |
| 3.333 | 2.327 | 12.66 | 4.679 | 1.254 | 0.2679 |
| 2.500 | 6.344 | 17.78 | 8.317 | 3.418 | 0.4110 |
| 2.000 | 11.25 | 21.57 | 11.62 | 6.064 | 0.5218 |
| 1.667 | 16.15 | 24.40 | 14.38 | 8.702 | 0.6051 |
| 1.429 | 20.61 | 26.57 | 16.63 | 11.11 | 0.6678 |
| 1.250 | 24.52 | 28.27 | 18.46 | 13.21 | 0.7157 |
| 1.111 | 27.88 | 29.64 | 19.95 | 15.02 | 0.7529 |
| 1.053 | 29.38 | 30.23 | 20.60 | 15.83 | 0.7685 |
| | | A deterioration in the quality of $x_1$ ($b_1^o = 1.0$, $b_1^t = 0.5$) | | | |
| 10.00 | 0.0005572 | 2.489E-05 | −5.914E-05 | −0.0002 | 3.886 |
| 5.000 | 0.2937 | 0.07394 | −0.05506 | −0.1212 | 2.201 |
| 3.333 | 2.327 | 1.053 | −0.5569 | −0.9599 | 1.724 |
| 2.500 | 6.344 | 3.879 | −1.748 | −2.617 | 1.498 |
| 2.000 | 11.25 | 8.238 | −3.399 | −4.643 | 1.366 |
| 1.667 | 16.15 | 13.27 | −5.198 | −6.663 | 1.282 |
| 1.429 | 20.61 | 18.30 | −6.944 | −8.504 | 1.225 |
| 1.250 | 24.52 | 22.96 | −8.542 | −10.11 | 1.184 |
| 1.111 | 27.88 | 27.12 | −9.964 | −11.50 | 1.154 |
| 1.053 | 29.38 | 29.01 | −10.61 | −12.12 | 1.143 |

in the example; the nested CES is a highly restrictive preference ordering and different degrees of bias will result with different preference orderings. However, this nested CES example does demonstrate that the $\mathrm{CV}_x$ *can* be significantly smaller than the CV and that one cannot get any indication of how much smaller if one just estimates a *partial* demand system for the $x$ commodities.

*B. The Relationships between the Different Equivalent Variation Measures*

LEMMA 2.

$$\textit{Given separability},\ \mathrm{EV}_x = \hat{\mathrm{EV}}. \tag{35}$$

The proof is in the appendix.

Lemma 2 allows us to abstract from $d$-utility and express $\mathrm{EV}_x$ as the amount the individual's expenditures on $x$ would have to be increased or decreased in the current state, $\{p^o, b^o\}$, to make him indifferent between the current state and the proposed state with the constraint, $\{p^\iota, b^\iota, z^o\}$. Lemma 2 also shows that $\mathrm{EV}_x$ will almost never be the desired equivalent variation. $\mathrm{EV}_x$ will only be the desired welfare measure if the individual is constrained to consume the same vector of the other market commodities after the prices and attributes have changed, and if, in addition, payment or receipt must be in terms of expenditures on $x$. Individuals are *usually not* constrained in this way.

THEOREM 2.

$$\textit{Given separability},\ \mathrm{EV}(y_x^\iota) \geq \mathrm{EV}. \tag{36}$$

*Proof.* The proof is analogous to the proof of Theorem 1.

While the fact that $\mathrm{EV}(y_x^\iota) \geq \mathrm{EV}$ is symmetrically appealing given that $\mathrm{CV}_x \equiv \mathrm{CV}_x(y_x^o) \leq \mathrm{CV}$, this bound does not have great practical significance, because $\mathrm{EV}_x(y_x^\iota)$ usually cannot be estimated. It depends on expenditures on the $x$ commodities in the proposed state, $y_x^\iota$, and these are not observed and cannot be predicted from the *partial* demand system. What is of practical importance is the relationship between $\mathrm{EV}_x \equiv \mathrm{EV}_x(y_x^o)$ and the EV. But, as can be demonstrated with examples, $\mathrm{EV}_x$ bounds the EV neither from above nor from below.[12] The relationship between $\mathrm{EV}_x$ and EV cannot be signed for two separate and opposing reasons. $\mathrm{EV}_x$ assumes an additional constraint in the proposed state, namely $z = z^o$. This factor will, ceteris paribus, make $\mathrm{EV}_x \leq \mathrm{EV}$. However, everything else is not constant; $\mathrm{EV}_x$ is measured in terms of expenditures on $x$, whereas EV is measured in unrestricted money. This factor will, ceteris paribus, make $\mathrm{EV}_x \geq \mathrm{EV}$. The two factors work in opposite directions, and $\mathrm{EV}_x$ can be greater than, less than, or equal to the EV.

An immediate corollary to the theorem proving $\mathrm{EV}_x(y_x^\iota) \geq \mathrm{EV}$ is that if $z$ is empty (i.e., if the *partial* utility function includes all market commodities), $\mathrm{EV}_x(y_x^\iota) = \mathrm{EV}$. When $x$ includes all the market commodities, $y_x^\iota = y_x^o = y^o$. Therefore, $\mathrm{EV}_x = \mathrm{EV}$ when $z$ is empty.

[12]Symmetrically, $\mathrm{CV}_x(y_x^\iota)$ bounds the CV from neither above nor below. It is possible to prove that $\mathrm{CV}_x(y_x^\iota) + (y_x^0 - y_x^\iota) \leq \mathrm{CV}$.

## VI. SUMMARY AND POLICY IMPLICATIONS

The applied economist often assumes, either explicitly or implicitly, that the commodities of interest are separable from many of the other commodities in the preference ordering. Assuming a separable group allows the analyst to estimate the system of *partial* demand functions for the market commodities in that group with just existing data on the prices and consumption levels for those market commodities, the quantities of the nonmarket commodities in the group, and the budget allocation to the group. This system of *partial* demand functions, along with the current budget allocation to the group, can then be used to derive a *partial* compensating (and equivalent) variation associated with any proposed change in the prices of the group's market commodities and/or exogenous levels of nonmarket commodities. However, this "shortcut" method of obtaining consumer's surplus measures is not done without cost.

*Consider first all those cases where the analyst's separability assumption is appropriate.* The *partial* welfare measures ($CV_x$ and $EV_x$) are not, in general, equal to the desired conventional measures (CV and EV). They are only equal in a very special case. Given this, one must ask what information $CV_x$ provides about the CV and what information $EV_x$ provides about the EV.

When one defines a separable group, one excludes from that group either some market commodities, some nonmarket commodities, or both. In the typical application, the separable group will exclude both market and nonmarket commodities. Theorem 1 states that in any case where the separable group does not include all of the market commodities, $CV_x$ is only a lower bound on the desired CV. That is, the $CV_x$ is only a lower bound estimate of how much the individual will pay for an improvement, and an upper bound estimate, in absolute terms, of how much he will have to be paid to accept a deterioration. Therefore, if, for a specific individual, $CV_x > 0$ one can conclude that the proposed policy will make the individual better off, but $CV_x < 0$ does not indicate whether the policy will be an improvement or deterioration for this individual. Aggregating the $CV_x$ across individuals, $\Sigma_i CV_x > 0$ is sufficient, but not necessary, to fulfill a necessary condition for the policy to be a potential pareto improvement, but $\Sigma_i CV_x < 0$ provides no information in this regard. When the separable group does not include all of the market commodities, $EV_x$ bounds the EV from neither above nor below. Therefore, if the separable group does not include all the market commodities, $EV_x$ tells the analyst nothing about the desired EV and is effectively policy irrelevant. This is a strong negative result.

Continuing to assume the separability assumption is appropriate, consider a situation where the analyst estimates a demand system that includes all the market commodities as a function of the complete budget, all of the prices, and the quantities of the affected nonmarket commodities. The other nonmarket commodities are excluded. In this case, the distinction between the *partial* and *complete* demand system is quite subtle. Given it is correct to assume that the market commodities, and the policy affected nonmarket commodities, form a separable group, then $CV_x = CV$ and $EV_x = EV$. In this case, the *partial* demand system is effectively complete; i.e., it includes all the market commodities, and there is no distinction between the total budget and the budget allocation to the separable group.

*Now consider those cases where the separability assumption is not appropriate.* If the analyst assumes a separable group that excludes some of the market commodities but the exclusion is not appropriate, the "supposed" *partial* welfare measures will, in general, bound the CV and EV from neither above nor below. They are of no value. They are not *partial* welfare measures and they cannot be interpreted as complete measures.

Alternatively, assume the analyst assumes a separable group that includes all of the market commodities but excludes some of the nonmarket commodities. If this separability assumption is not appropriate, then whether the derived welfare measures have value depends on whether the excluded nonmarket commodities will remain at their current levels after the proposal is enacted. This case is also quite subtle. When the demand system includes all the market commodities but it is not correct to assume that the market commodities and the affected nonmarket commodities form a separable group, then the demand system is complete and one obtains, with some qualification, the CV and EV. Since separability is not appropriate, the parameters in the demand system imbed the influence of the omitted nonmarket commodities and will change values if the levels of these nonmarket commodities change. Therefore, the derived CV and EV are only correct if the quantities of the excluded nonmarket commodities remain at their original levels after the proposed policy is enacted. For example, they cannot be used to analyze policies if the weather is an omitted nonmarket commodity and the weather is expected to change. Therefore, when the estimated demand system includes all the market commodities but not all of the nonmarket commodities, the robustness of the derived welfare measures depends greatly on whether the separability is appropriate.

Concluding, given the distinction between the compensating (equivalent) variation and its corresponding *partial* variation, great care is needed when using estimated *partial* welfare measures to evaluate policy.

## APPENDIX

LEMMA 1. *Given separability,* $CV_x = C\hat{V}$ *(Eq. (32)).*

*Proof.*

$$\mathrm{CV}_x = y_x^o - m_d(p^\iota, b^\iota, u_d^o) \tag{20}$$

$$\mathrm{C\hat{V}} = y^o - \hat{m}(p^\iota, b^\iota, q^o, c^o, u^o, z^o), \tag{30}$$

but

$$y_x^o = y^o - y_z^o \qquad \text{by definition.} \tag{37}$$

Given (20), (37), and (30),

$$\mathrm{CV}_x - \mathrm{C\hat{V}} = \hat{m}(p^\iota, b^\iota, q^o, c^o, u^o, z^o) - m_d(p^\iota, b^\iota, u_d^o) - y_z^o. \tag{38}$$

Express $\hat{m}(p^\iota, b^\iota, q^o, c^o, u^o, z^o)$ as a function of $m_d(p^\iota, b^\iota, u_d^o)$,

$$\hat{m}(p, b, q, c, u, z) = \min_x \sum p_j x_j + \sum q_i z_i, \tag{39}$$

subject to

$$u = \phi[u_d(x, b), z, c]. \tag{10}$$

But $\min_x \sum p_j x_j$ subject to $u_d = u_d(x, b)$ equals

$$m_d(p, b, u_d) \qquad \text{by definition.} \tag{15}$$

Therefore,

$$\hat{m}(p, b, q, c, u, z) = \sum q_i z_i + m_d(p, b, u_d), \tag{40}$$

where

$$u = \phi[u_d, z, c]. \tag{10}$$

Or more specifically,

$$\hat{m}(p^\iota, b^\iota, q^o, c^o, u^o, z^o) = \sum q_i^o z_i^o + m_d(p^\iota, b^\iota, u_d^o), \tag{41}$$

where

$$u^o = \phi[u_d^o(y_x^o), z^o, c^o] \qquad \text{by definition.} \tag{42}$$

Substituting (41) into (38), one obtains

$$\begin{aligned} \mathrm{CV}_x - \mathrm{C\hat{V}} &= \sum q_i^o z_i^o + m_d(p^\iota, b^\iota, u_d^o) - m_d(p^\iota, b^\iota, u_d^o) - y_z^o \\ &= \sum q_i^o z_i^o - y_z^o = 0 \qquad \text{by definition of } y_z^o \text{ and } z^o. \quad \text{Q.E.D.} \end{aligned} \tag{43}$$

LEMMA 2. *Given Separability, $E\hat{V} = EV_x$ (Eq. (35)).*

*Proof.*

$$\mathrm{E\hat{V}} = \hat{m}(p^o, b^o, q^o, c^o, \hat{u}^\iota, z^o) - y^o \tag{31}$$

$$\mathrm{EV}_x = m_d(p^o, b^o, u_d^\iota(y_x^o)) - y_x^o, \tag{23}$$

but

$$y_x^o = y^o - y_z^o. \tag{37}$$

Therefore, given (23), (37), and (31),

$$\mathrm{EV}_x - \mathrm{E\hat{V}} = m_d(p^o, b^o, u_d^\iota(y_x^o)) - \hat{m}(p^o, b^o, q^o, c^o, \hat{u}^\iota, z^o) + y_z^o. \tag{44}$$

Express $\hat{m}(p^o, b^o, q^o, c^o, \hat{u}^\iota, z^o)$ as a function of $m_d(p^o, b^o, u_d^\iota(y_x^o))$. From (40) and (10)

$$\hat{m}(p^o, b^o, q^o, c^o, \hat{u}^\iota, z^o) = \sum q_i^o z_i^o + m_d(p^o, b^o, u_d^\iota(y_x^o)), \tag{45}$$

where

$$\hat{u}^\iota = \phi[u_d^\iota(y_x^o), z^o, c^o] \qquad \text{by definition.} \tag{46}$$

By substituting (45) into (44) one obtains

$$\begin{aligned} \mathrm{EV}_x - \mathrm{E\hat{V}} &= m_d(p^o, b^o, u_d^\iota(y_x^o)) - \Sigma q_i^o z_i^o - m_d(p^o, b^o, u_d^\iota(y_x^o)) + y_z^o \\ &= y_z^o - \sum q_i^o z_i^o = 0 \qquad \text{by definition of } y_z^o \text{ and } z^o \quad \text{Q.E.D.} \end{aligned} \tag{47}$$

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