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The Demand for Site-Specific Recreational Activities: A Characteristics Approach[1]

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A model of constrained utility maximizing behavior is developed to explain how a representative individual allocates his ski days among alternative sites. The physical characteristics of the ski areas and the individual's skiing ability are explicit arguments in the utility function; the budget allocation is given along with the parametric costs to ski (including travel costs, entrance fees, equipment costs, and the opportunity cost of his time). Shares (a site's share being the proportion of ski days that the individual spends at that site) are derived and assumed multinomially distributed, a stochastic specification which maintains the inherent properties of the shares. Maximum likelihood estimation confirms the basic hypothesis that costs, ability, and characteristics all are important determinants of the sites' shares. The model explains a large proportion of the skier's allocation of ski days. A multinomial logit model of skier behavior is also developed and maximum likelihood estimates of its parameters are obtained. Examination of the summary statistics from my model and the logit model indicates that my model predicts the skier's choice of sites better than the logit model.

1. INTRODUCTION

The purpose of this research is to model and estimate a representative individual's share functions for site-specific skiing activities. The model assumes utility-maximizing behavior: Faced with a limited skiing budget, the individual attempts to allocate his time among competing areas so as to maximize the utility he derives from skiing. This utility is hypothesized to be a function of: (1) the amount the individual skis at each of the available sites; (2) certain physical characteristics of those sites; and (3) his skiing ability, i.e., whether he is a novice, an intermediate, or advanced skier. The cost of skiing at a specific site is hypothesized to be the sum of: (1) the price of the lift ticket; (2) the cost of equipment rental; (3) vehicle transportation costs; and (4) the opportunity cost of the individual's time, both while traveling and skiing. A system of stochastic share equations is derived from this model, a site's share being the proportion of ski days that the individual decides to spend at that particular site. Maximum likelihood estimates are obtained by applying the model to a cross-sectional sample of Colorado skiers.

[1]This paper has been evolving gradually. In part, it is based on my Ph.D. dissertation (University of British Columbia, 1978) and Discussion Paper 03/78, The Norwegian School of Economics and Business Administration (presented at the 1978 Econometric Society meetings in Geneva). After the model was substantially expanded it was revised and appeared as Resources Paper No. 33 in the University of British Columbia's Programme in Natural Resource Economics. The Programme's funding by the Social Sciences and Humanities Research Council of Canada is gratefully acknowledged. Computer funding provided by the Economics Department at U.B.C. is also gratefully acknowledged. Since then the paper has been substantially expanded into its present form. I am most indebted to my dissertation supervisor, J. G. Cragg, and other committee members, H. F. Campbell, A. D. Woodland, and T. G. Wales. Thanks for helpful suggestions and constructive comments must also go to G. C. Archibald, W. E. Diewert, J. Mossin, and A. Sandmo.

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This model of skier behavior developed here builds on and expands on the models of recreational demand developed by Burt and Brewer [4], and Cicchetti *et al.* [7]. The Burt and Brewer [4] model, and the Cicchetti *et al.* [7] model both made significant contributions to the art of recreational demand estimation but the demand equations in these models are not derived explicitly from a well-specified model of consumer behavior. This has led to two problems: (1) the functional forms of the estimated demand equations are often theoretically implausible, or if plausible, they impose questionable restrictions on the individual's preferences; and (2) the costs of producing recreational activities are often incorrectly defined, since the production functions for those activities are not adequately specified. This paper makes a moderate attempt to reduce the magnitude of these deficiencies.

Few demand studies for recreational sites explicitly consider the sites' physical characteristics. Even those studies which consider site characteristics do not explicitly incorporate them into the utility function (see, for example, Burt and Brewer [4] or Grimes [16]). I include physical characteristics of the sites (i.e., amounts and types of terrain, lift capacities, and average annual snowfall) directly into the utility function because, a priori, it seems very reasonable that an individual's choice among ski areas is influenced by them. I have also hypothesized that an individual's preferences over sites depend on his skiing ability, in that it can remove certain ski runs from his feasible choice set. For example, a beginner cannot take advantage of the expert runs at a site, consequently one would not expect the acres of expert runs to play an important explanatory role in how that individual chooses among sites. The method used to incorporate the characteristics of areas and of individuals appears to be original.

The shares were assumed multinomially distributed, a stochastic specification consistent with the inherent properties of the shares. This error specification for the shares also appears to be new. In the Appendix my model is compared with a multinomial logit model, where I argue that mine is a preferable approach to the skier's choice problem.

Prices and characteristics were found to be significant explanatory variables. The estimated share equations were found to be consistent with the underlying theory, and a large proportion of the variation in the data was explained by the model. Elasticities estimating the effect of changes in prices and characteristics on a site's share were derived. Policy implications of these elasticities are briefly discussed.

2. A MODEL OF SKIER BEHAVIOR

The purpose of this section is to develop a model which describes how a representative individual allocates a fixed skiing budget among competing ski areas. First (Section 2.A) the complete model is presented. Then I backtrack and consider the utility function (Section 2.B) and the cost functions and resulting shadow prices for the skiing activities (Section 2.C).

2.A. The Representative Skier

A model is required which describes how the representative individual allocates a predetermined skiing budget among alternative sites.[2] The allocation is hypothesized

[2] The behavior of the representative individual is defined as the expected behavior of an individual randomly drawn from the population.

to depend in part on the parametric costs of skiing at different sites. The skier allocates his budget among sites so as to maximize the utility he receives from skiing given these costs. The utility produced by skiing activities is assumed weakly separable from the utility produced by other activities.[3] Therefore, the allocation of the skiing budget is independent of total income, prices and characteristics of nonskiing activities, and the preference ordering for those activities (Phlips [20, pp. 72–77]).

The model must explain what determines the utility an individual derives from skiing activities, i.e., what determines why he prefers some sites over others. This is hypothesized to depend in part on the amount and types of terrain at the different sites. Ski terrain is designed for specific ability levels, thus one's ability to enjoy an area depends on one's skiing ability in conjunction with the amounts of novice, intermediate, and advanced terrain at the site. The lift capacities and snowfalls at the sites are also hypothesized to affect utility. It is further assumed that the individual desires variety in his skiing activities, i.e., that the marginal utility from skiing at a site diminishes with increasing visits.

The model of skier behavior is based on three postulates.

POSTULATE 1. Skiing is an activity. The skier combines skiing equipment, the services of a ski area, transportation services, and some of his own time to produce a site-specific skiing activity.

POSTULATE 2. The arguments in the utility function are the amount the individual skis at each of the available sites and the effective physical characteristics of those sites. Each site, from the point of view of the individual, can be described in terms of five effective physical characteristics (EPCs). The adjectives "effective physical" refer to the fact that the EPCs depend on the physical characteristics of the site and the personal characteristics of the individual. The personal characteristics often determine which of the physical characteristics can be effectively utilized. The first EPC is the acres of ski terrain at the site which the individual is capable of skiing. The second is the acres specifically designed for the individual's skiing ability. It is assumed that the individual distinguishes between the different types of terrain he is capable of skiing. These first two EPCs depend on the individual's skiing ability (novice, intermediate, or advanced), and the acres of novice, intermediate, and advanced terrain at the site. The VTF (Vertical Transport Feet) at the site (the number of people that can be transported 1000 vertical feet by the lift system in one hour) is the third EPC. The fourth is the site's average annual snowfall. I expect these first four to be the important EPCs of the sites.[4] The fifth EPC is an index of other physical characteristics of the site that can be effectively utilized by the individual. The individual observes all five EPCs, but we are unable to observe and measure the fifth one. The value of each site's fifth EPC varies across individuals and its expected value is assumed zero.

[3]A function is weakly separable across groups if the marginal rate of substitution between any two variables belonging to the same group is independent of the value of any variable in any other group (Phlips [20, p. 63]).

[4]One might think of others, but I feel that these four are the most important. The empirical model considers the behavior of a group for whom these four EPCs largely describe the sites. For example, all the people sampled were single and mostly took one-day trips. Characteristics of e.g., child-care facilities, lodges, and nightlife are therefore not important. For more details see Footnote 14.

POSTULATE 3. The quantity of skiing activity j and site j's EPCs are assumed strongly separable from the amount consumed of any other skiing activity, and its EPCs.[5] This assumption is proposed as a reasonable working hypothesis, I do not expect it to hold strictly.[6] The assumption reduces the number of parameters to be estimated in any model consistent with Postulates 1 and 2. This proves to be of critical importance in my empirical work. The assumption also makes it possible to ascertain the effects of introducing a new area into the skier's choice set without having to reestimate the model. Strong separability is only tenable if the skier views each of the areas as a distinct and independent alternative. Differential substitutability and complementarity amongst areas is not allowed. It is sometimes possible for a skier to ski at more than one site on a given trip.[7] In such a situation the skier will tend to lump these sites together and consider a visit to this group as one possible alternative; it is therefore unreasonable to view such sites as distinct and independent. To maintain consistency with Postulate 3, one must do what the skier does, the complementary sites must be aggregated and then treated as a distinct alternative. This was done in the empirical model.

The rational skier solves the following problem:[8]

$$\underset{\text{w.r.t. } Y}{\text{Maximize}}\, U = U(Y, A), \tag{2.1}$$

$$\text{s.t. } \tau = \Gamma' Y, \tag{2.2}$$

where

$Y \equiv [y_j]$, where $y_j \equiv$ the amount of skiing activity j produced and demanded by the individual per season, where one unit of y_j is one day of skiing at site j.

$\Gamma \equiv [\gamma_j] \equiv$ the cost (measured in units of time) of skiing activity j. γ_j is the hours required to produce one day of skiing at site j. It includes skiing time, transportation time, and the time required to earn the money that is needed to purchase (or rent) the skiing equipment and the lift ticket.

$\tau \equiv$ the individual's total time allotment to skiing activities.

$A \equiv [a_{kj}]$, where $a_{kj} \equiv$ the amount of EPC k that the individual can utilize at site j. Specifically: $a_{1j} \equiv$ the acres of ski runs at site j which the individual is capable of skiing. For example, the intermediate skier is limited to the novice and intermediate terrain. Skiers are assumed incapable of skiing terrain which has a difficulty rating in excess of their ability level. $a_{2j} \equiv$ the acres of

[5] A function is strongly separable across groups if the marginal rate of substitution between two variables belonging to different groups is independent of the quantity of any variables in another group (Phlips [20, p. 69]).

[6] Strong separability is a maintained hypothesis in many empirical demand studies. Empirical multiple-choice logit models—a possible alternative way of modeling the skier's choice on the basis of prices and characteristics—also universally maintain this hypothesis. A logit model of skier behavior is considered in the Appendix.

[7] This possibility is limited in my model because I only consider one-day trips.

[8] This model is more general than the Lancaster model [18]. The Lancaster model assumes that the utility one receives from a characteristic is independent of which activity produced it. It assumes constant returns to scale in the production of characteristics, diminishing marginal utility associated with visiting a specific site is disallowed by assumption. Such restrictions are not imposed on (2.1).

ski runs at site j specifically designed for the individual's skiing ability. For example, a_{2j}, for an intermediate skier would be the acres of intermediate terrain.

$a_{3j} \equiv$ the vertical transport feet (VTF) at site j.

$a_{4j} \equiv$ the average annual snowfall at site j (measured in inches).

$a_{5j} \equiv$ the amount of EPC five (the index) at site j.

$E(a_{5j}) = 0$. One could also define $a_{6j} = a_{1j} - a_{2j}$, which is the acres of terrain at site j on which the individual can ski but that are not designed for the individual's skiing ability. a_{6j} adds no new information but later it will be useful in expressing some elasticities.

A specific form of the utility function was chosen so as to be simple and consistent with Postulates 1, 2, and 3.[9]

$$U = \sum_{j=1}^{J} y_j^{\beta} h(a_{1j}, a_{2j}, a_{3j}, a_{4j}, a_{5j}), \tag{2.3}$$

where

$$\begin{aligned} h(a_{1j}, a_{2j}, a_{3j}, a_{4j}, a_{5j}) = {} & \alpha_0 + \alpha_1 a_{1j} + \alpha_2 (a_{1j} a_{2j})^{1/2} + \alpha_3 a_{2j} + \alpha_4 a_{1j}^{1/2} \\ & + \alpha_5 a_{2j}^{1/2} + \alpha_6 a_{3j} + \alpha_7 (a_{1j} a_{3j})^{1/2} + \alpha_8 (a_{2j} a_{3j})^{1/2} \\ & + \alpha_9 a_{3j}^{1/2} + \alpha_{10} a_{4j} + \alpha_{11} (a_{1j} a_{4j})^{1/2} + \alpha_{12} (a_{2j} a_{4j})^{1/2} \\ & + \alpha_{12} (a_{2j} a_{4j})^{1/2} + \alpha_{13} (a_{3j} a_{4j})^{1/2} + \alpha_{14} a_{4j}^{1/2} + \alpha_{15} a_{5j} \\ & + \alpha_{16} (a_{1j} a_{5j})^{1/2} + \alpha_{16} (a_{2j} a_{5j})^{1/2} + \alpha_{17} (a_{3j} a_{5j})^{1/2} \\ & + \alpha_{18} (a_{4j} a_{5j})^{1/2} + \alpha_{19} a_{5j}^{1/2}. \end{aligned} \tag{2.4}$$

The parameters in (2.3) are $\alpha_0, \alpha_1, \alpha_2, \ldots, \alpha_{19}$, and β. The following system of share equations are obtained by maximizing the utility function (2.3) subject to the budget constraint (2.2). These shares, which express the proportion of ski days to a given site, are:[10]

$$s_j^* \equiv y_j^*/T^* = 1 \Big/ \sum_{k=1}^{J} \left[\frac{\gamma_k h(a_{1j}, a_{2j}, a_{3j}, a_{4j}, a_{5j})}{\gamma_j h(a_{1k}, a_{2k}, a_{3k}, a_{4k}, a_{5k})} \right]^{-\sigma}, \qquad j = 1, \ldots, J, \tag{2.5}$$

where $T^* \equiv \sum_{k=1}^{J} y_k^*$ and $-\sigma = 1/(\beta - 1)$.

$\sigma = 1/(1-\beta)$

[9]The utility function for the representative individual is the expected value of (2.3) and (2.4). Keep in mind that $E(a_{5j}) = 0$. It is possible to rewrite (2.3) and (2.4) in the form:

$$\begin{aligned} U &= U(Y, A) \\ &= V(Y, a_1, a_2, a_3, a_4) + \epsilon(Y, a_1, a_2, a_3, a_4, a_5), \end{aligned}$$

where $V(Y, a_1, a_2, a_3, a_4)$ reflects the tastes of the representative individual in the population; it is nonstochastic, and $\epsilon(Y, a_1, a_2, a_3, a_4, a_5)$ reflects the variations in the utility the individuals receive from the different sites due to the fact that a_5 varies across individuals and sites.

$$E[\epsilon(Y, a_1, a_2, a_3, a_4, a_5)] = 0.$$

[10]The system of demand equations ($y_j^* = 1, 2, \ldots, J$) corresponding to the preference ordering (2.3) is reported by Pollak [21, p. 403]. The share equations are derived by dividing y_j^* by the sum of the y_k^*. Many of the terms, including the skiing budget, τ, cancel.

All the share functions (2.5) are identical. The only thing that varies from one site's share function to another is the value of the exogenous variables $(\gamma_j, a_{1j}, a_{2j}, \ldots, a_{5j})$. If two sites ($j$ and k) are identical, that is, if $\gamma_j = \gamma_k$ and $a_{lj} = a_{lk}$, $l = 1, 2, \ldots, 5$, then s_j^* will equal s_k^*. Share functions are generally not identical because not all factors (i.e., characteristics) that explain variations in the demand for activities are included as exogenous variables. Therefore, variations in demand across activities have to be, at least partially, accounted for by variations in the form of the share equations. In the case of skiing there are only six reasons why the demand for one skiing activity might differ from the demand for another: their prices differ, or they possess different quantities of EPCs. Since all are explicitly included as arguments in the share function, all the functions are identical. An activity's name (site name) is not necessary to explain the demand for that activity. This is the advantage gained when one explicitly includes the EPCs of activities as arguments in the utility function. One can account for differences in the demand for activities by variations in the values of the independent variables (the prices and the EPCs) in the share function, rather than having the variations appear in the form of differing share functions, each specific to only one name specific activity.

2.B. *The Specific Form of the Utility Function*

The utility function (2.3) belongs to the Bergson family of utility functions, that is, utility functions which are both directly additive and homothetic (see Samuelson [25, pp. 787–788] and Pollak [25, pp. 787–788]).[11] Homothetic preferences mean that the sites' share are insensitive to changes in the skiing budget (τ). This restrictive assumption was made for practical reasons. The indifference maps corresponding to the preference ordering (2.3) are identical to the isoquant maps of the CES production function (see Chipman [5, pp. 485] and [6, pp. 57–70]), and the Allen [1] elasticity of substitution between any two skiing activities is constant and equal to:

$$\sigma_{jk} = \sigma = -1/(\beta - 1), \qquad j \neq k,\ j, k = 1, \ldots, J \text{ (Uzawa [28])}. \tag{2.6}$$

$\sigma = 1/(1-\beta)$

The constant terms (h_j, $j = 1, \ldots, J$) in the Bergson function ($U = \sum_{j=1}^{J} h_j y_j^{\beta}$) were disaggregated and each made an identical function of the EPCs of site j ($h_j = h(a_{1j}, \ldots, a_{5j})$). This technique for incorporating the EPCs into the direct utility function is similar to the method used by Pollak and Wales [22] to incorporate demographic characteristics of the household directly into the household's utility function. Pollak and Wales assumed that a subset of the parameters in the indirect function was a function of demographic variables which depend only on the individual, whereas mine depend on both the sites and the individual. Incorporating both characteristics of the individual and characteristics of the areas into a conventional utility function using this method appears to be new. The specific form for $h(a_{1j}, \ldots, a_{5j})$, defined by (2.4), was chosen because it is a second order approximation to any twice differentiable function in the a_{kj}'s.

[11] A function is homothetic in the y_j's if it can be written: $U = G[g(y_1, \ldots, y_J)]$ (Phlips [20, pp. 36–87]), where G is a finite, continuous and strictly monotonically increasing function of one variable with $G(0) = 0$, and where g is a homogeneous function of the J variables, $y_1, \ldots, y_J$.

2.C The Cost Functions for the Skiing Activities

The budget constraint is described by (2.2), where the prices of the J skiing activities are assumed parametric to the individual. Four nonsubstitutable inputs are required to produce a one-day ski trip to site j: (1) one day's use of ski area j; (2) skiing equipment (skis, boots, etc.); (3) transportation services to and from site j; and (4) the time required to ski, and to travel to and from the site. The fact that these inputs do not substitute for one another suggests that a Leontief process will closely approximate the "true" production function for a skiing activity. Thus it is assumed that the price (marginal cost measured in units of time) of skiing activity j is parametric to the individual and equals:

$$\gamma_j = \frac{\begin{pmatrix}\text{lift ticket}\\ \text{price at}\\ \text{site } j\end{pmatrix} + \begin{pmatrix}\text{ski}\\ \text{equipment}\\ \text{rental fee}\end{pmatrix} + b_j \begin{pmatrix}\text{per mile}\\ \text{transporta-}\\ \text{tion costs}\end{pmatrix}}{w} + c_j, \qquad (2.7)$$

where

$b_j \equiv$ the minimum number of miles the individual must travel to produce one unit of skiing at site j, i.e., twice the distance from the individual's residence to site j.
$w \equiv$ the opportunity cost (measured in \$) of the individual's time.
$c_j \equiv$ the minimum amount of time required by the individual to ski and to travel to and from site j.

3. THE STOCHASTIC ASPECTS OF THE MODEL

One would like to determine the value of the vector of parameters, θ, $\equiv$ $[\alpha_0, \alpha_1, \ldots, \alpha_{19}, -\sigma]$ in the system of share equations derived in the last section.

$$s_j^* = s^*(\gamma_j, a_{1j}, a_{2j}, a_{3j}, a_{4j}, a_{5j}; \Gamma; A; \theta), \qquad j = 1, \ldots, J. \qquad (3.1)$$

The model could then be used to determine the proportion of ski days that the individual will spend at site j—the shares. If, given a sample of individuals, one could observe the values of all the independent and dependent variables, then one could deterministically solve the system of equations to obtain the values of the parameters. Unfortunately, the problem is not so simple because the value of the fifth EPC, at each site and for each individual, is not observed. The observed shares, as a function of the observed costs (Γ) and observed EPCs (a_1, a_2, a_3, a_4), are therefore stochastically distributed. One can then only obtain statistical estimates of $[\alpha_0, \alpha_1, \ldots, \alpha_{13}, \alpha_{14}, -\sigma]$. The parameters relating to the fifth EPC ($\alpha_{15}, \ldots, \alpha_{19}$) cannot be estimated. The distribution of the shares in the sample depend on the distribution of a_5. I have assumed for simplicity that $E(a_{5j}) = 0$. There is not enough information to completely specify a_5's density function explicitly aprior.

The individual solved his utility maximization problem implicitly assuming that ski trips are completely divisible, but they are not. Ski trips must be consumed in one-day increments. The individual must adjust his desired trip vector to take account of the lumpy nature of the choice problem. This adjustment process is unknown but will most likely vary among individuals. These variations in the adjustment process will add a second stochastic component to the observed share equations.

Estimation of the parameters $[\alpha_0, \alpha_1, \ldots, \alpha_{14}, -\sigma]$ requires that we specify the density function for the stochastic component of the shares. This density function depends on the distribution of a_5 across individuals and on the different ways the individuals adjust to the lumpy nature of the choice problem. Its specific form is unknown so we are required to approximate it on the basis of its necessary properties. Our definition of the share, $s_j \equiv y_j/T$, requires that it can take only one of $(T+1)$ discrete values in the 0–1 range, where $\Sigma_{j=1}^{J} s_j = 1$. This follows because ski trips can only be consumed in integer (day) increments and because $T = \Sigma_{j=1}^{J} y_j$. Each share is also perfectly correlated with the other $J-1$ site-specific shares. Our prior assumption that $E(a_{5j}) = 0$ implies that $E(s_j) = s_j^*$.

A standard assumption in econometric work is that the random variable is normally distributed. Unfortunately this approximation is inconsistent with the properties of the s_j's. The normality assumption requires that the random variable be continuously distributed from $-\infty$ to $+\infty$, where there is a positive probability that shares will be outside the 0–1 range. This is inconsistent with the requirement that each s_j is discretely distributed between zero and one. The normal distribution also assumes that the shares are symmetrically distributed. This seems unlikely for shares with expected values near zero or one.[12] Even when the population is not normally distributed, the normal distribution can often be justified as the appropriate density function by an appeal to the Central Limit Theorem. Unfortunately, the average skier does not ski enough times in a season to invoke this justification for the use of the normal density function.[13] The standard normality assumption must thus be rejected.

It is assumed that the individual's density function for s_j, $j = 1, \ldots, J$, is a multinomial, where:[14]

$$f(s_1, s_2, \ldots, s_j; T; \theta) = \left(T! \Big/ \prod_{j=1}^{J} y_j! \right) \left(\prod_{j=1}^{J} (s_j^*)^{y_j} \right). \tag{3.2}$$

This error specification for a system of share equations, where the shares are not probabilities associated with choosing a site on a given trip, appears to be new. In the Appendix I compare it with a multinomial logit model. There it is argued that my model is preferable for the skiers' choice problem. Wilks [29, p. 139], amongst others, have shown that, if the s_j are distributed as a multinomial, then:

$$E(s_j) = s_j^*, \qquad j = 1, \ldots, J, \tag{3.3}$$

$$\operatorname{var}(s_j) = (s_j^*)(1 - s_j^*)/T, \qquad j = 1, \ldots, J, \tag{3.4}$$

$$\operatorname{cov}(s_j s_k) = -(s_j^*)(s_k^*)/T, \qquad j \neq k, k = 1, \ldots, J. \tag{3.5}$$

[12] Discussions with A. D. Woodland, who is also examining the inappropriateness of the normality assumption with system of share equations [30], helped to clarify some of my thoughts on these matters.

[13] The average individual in my sample took only nine ski trips during the season.

[14] I am not assuming that s_j^* is the probability that site j will be chosen on a given trip, and I am not assuming that each trip, for a given individual, is independent. This is contrary to the normal interpretation of the variables in the multinomial distribution, but this does not preclude me from utilizing the mathematical properties of the function given that I fulfill the requirements that

$$1 > s_j^* > 0, \qquad \sum_{j=1}^{J} s_j^* = 1, \qquad \text{and} \qquad \sum_{j=1}^{J} y_j = T.$$

The distribution of the s_j will be skewed, except in the case where $s_j^* = 1/J \forall j$. The variance of s_j falls toward zero as s_j^* approaches either its upper limit of one, or its lower limit of zero. As the number of trips increases, the variances and covariances of the s_j decrease. The covariance matrix satisfies the condition that $\Sigma_{k=1}^{J}\text{cov}(s_j s_k) = 0$, where the signs on all the covariances ($j \neq k$) are required to be negative.

The multinomial was chosen as an appropriate density function for the individual's shares because it is simple, and because it is consistent with all the aforementioned properties of the density function of s_j. The multinomial places the following required restrictions on the random variable s_j: (1) the expected value of s_j is s_j^*; (2) s_j is limited to $(T + 1)$ discrete values, all of which are in the 0–1 range; and (3) the s_j are correlated across sites in such a way that $\Sigma_{j=1}^{J} s_j = 1$.

If it is assumed that the choice of shares by one individual is completely independent of any other individual's choice, then the likelihood function for a sample of N skiers is:

$$L = \prod_{i=1}^{N} f(s_{1i}, s_{2i}, \ldots, s_{Ji}; T_i; \theta). \tag{3.6}$$

The i subscript refers to the ith individual, where $i = 1, \ldots, N$. The maximum likelihood estimate of the parameters for a particular sample is the $\hat{\theta}$ which globally maximizes the likelihood function (3.6). Rao [24, pp. 295–296] has shown that if certain regularity conditions are fulfilled, the maximum likelihood estimates of the parameters in (3.1) will be consistent and asymptotically efficient when s_j is multinomially distributed. Equation (3.1) will fulfill these regularity conditions for most populations.

4. DATA

Estimation of the share equations for J site-specific skiing activities requires four types of data: (1) a cross-sectional survey of skiers which details their skiing activities for an entire season at the J sites; (2) data on the acres of novice, intermediate, and advanced terrain at each of the sites; (3) data on vertical transport feet and snowfall at each of the sites; and (4) price data (lift ticket prices, transportation costs, etc.).

4.A. A Cross-Sectional Survey of Skiers

The best available skier survey is one done in 1968 by the Denver Research Institute (DRI). This survey was part of an extensive analysis of the Colorado tourist market (DRI [10]). The following data were collected for each individual sampled: (1) a complete record of each individual's skiing activities during the 1967/1968 season; (2) the individual's skiing ability; (3) information on the individual's family and their skiing habits; (4) the individual's occupation and approximate earning ability; and (5) the location of the individual's residence. It should be noted that the question ascertaining skiing ability did not classify a skier's ability as necessarily equivalent to the type of terrain he most enjoyed, but rather equivalent to the type of terrain he is capable of navigating.

TABLE I
Allocation of Student Ski Days by Ability Level

	Total	Novice	Intermediate	Advanced
Aspen	297	9	101	187
Vail	241	13	70	158
A-Basin	167	4	65	98
Breckenridge	96	15	38	43
Loveland	137	16	73	48
Winter Park	221	20	81	120
Broadmoor	7	1	5	1
Crested Butte	36	0	26	10
Lake Eldora	89	6	40	43
Monarch	43	4	15	24
Mt. Werner	69	0	43	26
Wolf Creek	10	0	4	6
Purgatory	20	0	4	16
Cooper	10	6	3	1
Hidden Valley	10	1	4	6
Total	1453	95	572	787
Average no. of ski days per season	8.9	6.79	7.94	10.22

My estimation of the share equations will be based on a subsample of the skiers questioned by DRI. This subsample will be restricted to include only single post-secondary Colorado students, who do not belong to a ski club and whose family does not own a dwelling at a ski area. There are 163 individuals in this group. Each of the skiers in the sample attends school (resides) in one of the following eleven Colorado cities: Denver, Boulder, Ft. Collins, Greeley, Golden, the Air Force Academy, Colorado Springs, Pueblo, Alamosa, Gunnison, and Durango. Their ski trips were limited almost exclusively to the following 15 distinct ski areas: Aspen (consisting of Aspen Highlands, Aspen Mountain, Buttermilk, and Snowmass), Vail, Arapahoe-Basin, Breckenridge, Loveland, Winter Park, Broadmoor, Crested Butte, Lake Eldora, Monarch, Mount Werner (Steamboat Springs), Wolf Creek, Purgatory, Cooper, and Hidden Valley (Estes Park). Table I summarizes the skiers' trips.[15]

[15] There were a number of reasons for limiting my sample to this group. (1) The technology I specified for producing skiing activities most accurately describes the production of one-day trips, and this is what students predominantly take (DRI [10, p. 74]). (2) I have modelled the behavior of the individual, not the family. Estimation is made simpler if one can assume that the individual's choice of sites is independent of any other individual's choice. Single students tend to ski without other members of their families (DRI [12, p. 6]). (3) The value of each individual's time must be estimated so that the costs of visiting each of the sites by each of the individuals can be determined. The DRI data include annual income data, but not hourly wage rates. It is questionable whether income data alone are sufficient to construct reliable estimates of the value of each individual's time. The alternative to constructing such an opportunity cost variable which varies across individuals, is to choose a subgroup of the population within which it is not unreasonable to assume that every individual's time has the same dollar value. The opportunity cost of time should be relatively stable across single postsecondary Colorado students. It can hopefully be approximated by using the 1968 hourly U.S. Federal minimum wage rate, which was $1.15 an hour (U.S. Department of Commerce [27, p. 382]). (4) Other questions in the survey suggest that postsecondary students, more so than other skiers, visit an area predominantly to ski. Students in my subsample ski approximately six hours per day (DRI [10, p. 75]). (5) It was simple to calculate travel costs for these students because they traveled almost exclusively by car (DRI [11]). (6) Many variables that possibly influence the choice of sites but that are not explicitly included as independent variables in the share equations, vary little within the student group.

4.B. Lift Ticket Prices and Characteristics of the 15 Colorado Ski Areas

Estimation of the share equations for the 15 ski areas requires that we know their lift ticket prices; the amounts of novice, intermediate and advanced terrain at each during the 1967/1968 season; vertical transport feet at each in 1967/1968; and average annual snowfall (measured in inches). These data are listed in Table II.

4.C. Construction of Cost and Effective Physical Characteristic Data for the Ski Areas

The vector of shadow prices for the 15 ski areas was calculated for each individual, where these shadow prices, γ_{ji}, are defined by (2.7). Distances were calculated using a Rand McNally road atlas [23]. The opportunity cost of the students' time was assumed to be $1.15/hr. as per assumption in Footnote 14. The student is assumed to ski six hours per day as per the information noted in Footnote 14. It was assumed that the average driving speed was 45 miles per hour. The 1967/1968 lift ticket prices are included in Table II. The rental fee for skiing equipment averaged $3.50 per day during the 1967/1968 season (Colorado Visitors Bureau [9]). In the previous section it was noted that the ski trips were made almost exclusively by automobile. The per mile variable cost of operating an automobile in 1968 was $0.064 (U.S. Department of Commerce [26, p. 537]). In my sample there were 3.8 members in each skiing party, so average per mile transportation costs for each individual was $0.017 per mile. This combination of further assumptions and additional information allows us to make individual i's cost function for ski activity

TABLE II

Lift Ticket Prices and Ski Area Terrain, VTF, and Snowfall, 1967/1968 Season

Ski areas	Prices	Acres novice terrain	Acres inter-mediate terrain	Acres advanced terrain	VTF	Snowfall
Aspen	6.50	624	1559	722	19222	233
Vail	7.00	1024	3072	1024	8849	301
A-Basin	4.75	100	160	140	2913	280
Breckenridge	5.00	70	140	140	4100	285
Loveland	5.00	122	220	73	4512	280
Winter Park	5.00	127	258	59	5130	250
Broadmoor	3.00	12	4	4	480	40
Crested Butte	5.00	98	24	35	1607	210
Lake Eldora	4.00	22	70	16	1484	150
Monarch	3.50	20	65	15	192	354
Mt. Werner	5.00	70	160	29	2914	325
Wolf Creek	3.00	10	20	17	384	435
Purgatory	4.50	25	25	50	1507	300
Cooper	2.75	86	108	22	856	250
Hidden Valley	3.50	10	16	50	1106	150

Sources: Lift Ticket Prices: Colorado Visitors Bureau [9]. Terrain Data: Data on the terrain at all of the areas except Aspen Corporation areas, Hidden Valley, Loveland and Wolf Creek was provided directly by the ski are a managements at my request. Estimates for the other areas were constructed on the basis of data provided by Colorado Ski Country U.S.A., Denver, Colorado. VTF: Colorado Ski Country U.S.A. (1974). Average annual snowfall: Colorado Ski Country U.S.A. (1975).

j more explicit.

$$\gamma_{ji} = \frac{\left[\begin{pmatrix}\text{lift ticket} \\ \text{price at} \\ \text{site } j\end{pmatrix} + (\$3.50) + b_{ji}(\$0.017)\right]}{\$1.15} + \frac{b_{ji}}{45} + 6. \tag{4.1}$$

The effective physical characteristics, a_{1ji} and a_{2ji}, were calculated for each site for each individual using the data on terrain and skiing ability.

5. EMPIRICAL RESULTS AND THEIR INTERPRETATIONS

In this section the maximum likelihood estimates of the parameters are reported and discussed. Hypothesis tests are performed to ascertain whether prices and characteristics play a statistically significant role in the skier's allocation of ski days among sites. The estimated preference orderings are also examined to determine if they are consistent with the underlying hypothesis of utility maximizing behavior. Elasticity estimates are reported to provide insights into the allocational behavior implied by the estimated model. Policy implications are also briefly discussed.

5.A. Maximum Likelihood Estimates and Hypothesis Testing

The DRI sample of student skiers was used to obtain estimates of the parameter vector $\theta \equiv [\theta_r] \equiv [\alpha_0, \alpha_1, \ldots, \alpha_{14}, -\sigma]$ in the model. Maximum likelihood estimates were obtained by finding those values of θ which maximize:

$$l^* = \sum_{i=1}^{163} \sum_{j=1}^{15} y_{ji} \log(s_{ji}^*), \tag{5.1}$$

where

$$s_{ji}^* = s^*(\gamma_{ji}, a_{1ji}, \ldots, a_{4ji}; \Gamma_i; A : \theta). \tag{3.1}$$

Equation (3.1) is homogeneous of degree zero w.r.t. the α parameters so the maximum likelihood estimates are not uniquely identified. To rectify this situation, α_0 was set equal to one. (5.1) was then maximized using a modified Newton method formulated by Fletcher [15] and supported by the U.B.C. Computer Center (Bird and Moore [2, p. 2]).[16]

Maximum likelihood estimates of the parameters were calculated for six models. These estimates along with the approximations to their corresponding asymptotic t statistics are reported in Table III. The basic hypothesis of this research is that prices and effective physical characteristics play an important role in the skier's allocation. The corresponding null hypothesis is that prices and characteristics play no role, i.e.,

[16] The algorithm used calculated the derivatives of l^* w.r.t. the parameters numerically rather than analytically. The asymptotic t statistics (reported in Table III) are therefore subject to approximation error.

TABLE III
Maximum Likelihood Estimates for Six Models

Model	l^*	α_1	α_2	α_3	α_4	α_5	$-\sigma$	α_6
1	−4021.454							
2	−4017.232						−0.2879	
							(−2.919)	
3	−3563.518	0.0269			−0.3299		−0.3447	
		(39.80)			(−86.11)		(−26.31)	
4	−3472.695	−0.1313	0.3713	−0.2815	3.210	−1.396	−2.128	
		(−2.905)	(2.837)	(−2.818)	(3.111)	(2.781)	(20.41)	
5	−3443.358	−0.0155	0.0414	−0.0294	0.3982	−0.1393	−2.025	0.0005
		(−1.792)	(1.719)	(−1.665)	(2.070)	(−1.220)	(−11.91)	(1.535)
6	−3410.214	−0.6144	1.445	−1.095	−3.164	−10.70	−1.984	−0.0165
		(−2.681)	(2.105)	(−2.130)	(−2.840)	(−10.08)	(−10.61)	(−1.627)

Model	α_7	α_8	α_9	α_{10}	α_{11}	α_{12}	α_{13}	α_{14}
5	0.0015	−0.0033	0.0082					
	(0.9756)	(−1.584)	(0.4605)					
6	0.0945	−0.1058	21.95	2.464	1.137	0.5934	−1.124	−48.209
	(2.388)	(−2.110)	(25.58)	(39.19)	(3.977)	(2.104)	(−19.70)	(−188.7)

the individual randomly allocates his ski days amongst sites, such that:

$$s_{ji}^* = 1/J = 1/15 \qquad \forall i \text{ and } j. \tag{5.2}$$

Equation (5.2) is a nested hypothesis of the model (3.1). If it is assumed that $\alpha_{.}-\alpha_{14} = -\sigma = 0$, then (3.1) reduces to (5.2). The log of the likelihood function, l^*, for this restricted case of the model (model 1—the null hypothesis) is −4021.454. The model is first made less restrictive by allowing prices, but not characteristics, to play an explanatory role in the skier's allocation amongst sites (model 2). This can be accomplished by determining the maximum likelihood estimate of $-\sigma$, given the restriction that $\alpha_1 - \alpha_{14} = 0$. The model is then generalized by allowing only effective physical characteristic a_{1ji} to play an explanatory role in the skier's choice of sites. This hypothesis (3) is that $\alpha_2 = \alpha_3 = \alpha_5 - \alpha_{14} = 0$. Model 4 includes the effects of only prices, a_{1j}, and a_{2j}, therefore $\alpha_6 - \alpha_{14} = 0$. Model 5 includes all the explanatory variables except a_{4j}, therefore $\alpha_{10} - \alpha_{14} = 0$. The full model (6) includes the effects of prices and all four EPCs. On the basis of likelihood ratio tests the full model (6) predicts the allocation of the skier's budget significantly better (.005) than model 5, which predicts better than 4, which predicts better than 3, which predicts better than 2, which predicts better than 1. Prices and the four EPCs of the sites play a significant role in how the skier allocates his ski days amongst the alternative sites.[17]

[17]It is a hypothesis of the model that all the relevant EPCs have been included. This hypothesis could be tested, in theory, by including fourteen dummy variables, one for each site, in the h function (2.4), and then estimating the full model. The coefficients on the dummies would measure those specific site effects not accounted for by the prices and the four included EPCs. The maintained hypothesis is therefore that the inclusion of the dummies will not significantly increase the explanatory power of the model. This hypothesis cannot be tested because the full model is then too large to estimate (28 parameters).

A modified R^2 can be calculated to give us an indication of the model's goodness of fit.

$$\bar{R}^2 = 1 - e^{(2[l^*_{H_0} - l^*_H]/T)}/1 - e^{(2l^*_{H_0}/T)} \qquad \text{Baxter and Cragg} \quad [3, \text{p. } 230], \tag{5.3}$$

where

$l^*_{H_0}$ = the log of the likelihood function if the null hypothesis is correct. $l^*_{H_0} = -4021.451$.
l^*_H = the log of the likelihood function if the hypothesis is correct. $l^*_H = -3410.214$.
T = the number of observations = 1453 ski trips.

$$\bar{R}^2 = 0.57. \tag{5.4}$$

It is also of interest to examine the actual and predicted shares for the 15 sites.

The estimated model is consistent with the hypothesis of utility maximizing behavior for each individual in the sample. For each individual i, the utility function $U(\hat{Y}_i A_i)$, is: (1) continuous, finite, and quasi concave in $\hat{Y}_i \gg 0_{15}$; and (2) $\partial U/\partial y_{ji} > 0 \forall j$.

Conventional theory has suggested and empirical testing has confirmed that prices play an important explanatory role in the consumer's allocational behavior. But, characteristics of the activities and the consumer's ability to utilize those characteristics, also help to explain allocational behavior. Too often this hypothesis remains untested. Table V lists the predicted shares. The vectors of predicted shares vary across individuals in my sample because of variations in skiing ability and dispersion in the location of residence.

A number of allocational patterns can be discerned. First, an individual's predicted share for a specific site decreases as the distance increases between the site and his residence. The sites' share elasticities with respect to a change in the distance to the sites are all negative, falling in the range of 0 to -1. This property was first recognized by Clawson [7] and now forms the basis of the travel-cost technique. This result is not unexpected considering that automobile operating costs and the value of the individual's time while traveling are the major variable components of skiing costs at the different sites.

TABLE IV
Actual and Predicted Shares

	Aspen	Vail	A-Basin	Breckenridge	Loveland	Winter Park	Broadmoor	Crested Butte	Lake Eldora	Monarch	Mt. Werner	Wolf Creek	Purgatory	Cooper	Hidden Valley
Actual share	0.20	0.16	0.12	0.07	0.09	0.16	0.01	0.02	0.06	0.03	0.05	0.01	0.01	0.01	0.01
Predicted share	0.21	0.16	0.09	0.08	0.13	0.13	0.03	0.02	0.04	0.02	0.04	0.02	0.01	0.02	0.01
% Bias	+1%	0	−3%	+1%	+4%	−3%	+2%	0	−2%	−1%	−1%	+1%	0	+1%	0

TABLE V

$\hat{s}_{ji}$, The Predicted Shares

	$J =$	Aspen	Vail	A-Basin	Brecken-ridge	Love-land	Winter Park	Broad-moor	Crested Butte
	N	0.1731	0.1395	0.0851	0.0645	0.1260	0.1702	0.0590	0.0194
Denver	I	0.1480	0.1171	0.1056	0.0819	0.1539	0.1631	0.0391	0.0178
	A	0.2208	0.1932	0.0957	0.0816	0.1242	0.1195	0.0173	0.0112
	N	0.1727	0.1380	0.0833	0.0634	0.1231	0.1667	0.0507	0.0187
Boulder	I	0.1484	0.1164	0.1038	0.0808	0.1511	0.1605	0.0275	0.0173
	A	0.2209	0.1917	0.0939	0.0803	0.1217	0.1173	0.0149	0.0108
Ft.	N	0.1867	0.1413	0.0798	0.0619	0.1170	0.1604	0.0507	0.0202
Collins	I	0.1609	0.1194	0.0997	0.0792	0.1439	0.1549	0.0276	0.0187
	A	0.2371	0.1947	0.0893	0.0779	0.1147	0.1121	0.0148	0.0116
	N	0.1862	0.1420	0.0810	0.0627	0.1188	0.1626	0.0545	0.0208
Greeley	I	0.1599	0.1196	0.1008	0.0798	0.1457	0.1565	0.0296	0.0192
	A	0.2361	0.1954	0.0905	0.0787	0.1163	0.1135	0.0159	0.0119
	N	0.1703	0.1402	0.0878	0.0661	0.1305	0.1753	0.0483	0.0184
Golden	I	0.1447	0.1169	0.1083	0.0833	0.1583	0.1669	0.0259	0.0168
	A	0.2159	0.1929	0.0982	0.0830	0.1278	0.1223	0.0141	0.0105
Air Force	N								
Academy	I	0.1694	0.1112	0.0894	0.0809	0.1289	0.1390	0.0727	0.0268
	A	0.2540	0.1844	0.0815	0.0810	0.1046	0.1023	0.0396	0.0169
Colorado	N								
Springs	I	0.1755	0.1162	0.0828	0.0856	0.1193	0.1288	0.0810	0.0279
	A	0.2615	0.1915	0.0750	0.0852	0.0961	0.0942	0.0438	0.0175
	N	0.2096	0.1314	0.0617	0.0613	0.0898	0.1244	0.1249	0.0359
Pueblo	I	0.1850	0.1138	0.0789	0.0803	0.1131	0.1231	0.0697	0.0340
	A	0.2746	0.1868	0.0712	0.0796	0.0908	0.0897	0.0376	0.0212
	N								
Alamosa	I	0.1986	0.1315	0.0764	0.0764	0.0979	0.0922	0.0274	0.0402
	A								
	N	0.2425	0.1537	0.0602	0.0597	0.0782	0.0930	0.0436	0.1061
Gunnison	I	0.2036	0.1266	0.0734	0.0742	0.0937	0.0875	0.0231	0.0956
	A	0.2960	0.2036	0.0648	0.0721	0.0737	0.0625	0.0122	0.0584
	N								
Durango	I	0.2716	0.1063	0.0606	0.0590	0.0790	0.0783	0.0157	0.0487
	A	0.3745	0.1622	0.0507	0.0544	0.0589	0.0530	0.0078	0.0282

Examination of the predicted shares (Table V) and amounts of terrain and VTF at the sites (Table II) shows that the shares tend to be directly related to the size of the area in terms of terrain and VTF.

It is interesting that the shares depend on the level of the individual's skiing ability. Everyone is attracted to Aspen and Vail, but for all residential locations Aspen's and Vail's shares are highest for the advanced skier. Advanced skiers, relative to others, are attracted to areas with immense amounts of terrain, probably because they are the only ones with the ability and speed to ski the entire mountain. The advanced skier does not become bored with lack of variety of terrain. On the other hand, novice and intermediate skiers seem to like Winter Park and Loveland relatively more than advanced skiers do, probably because most of the terrain is designed for their ability levels (see Table II). Novices are more attracted to the smaller areas than are the intermediate and advanced skiers. This is probably due to the fact that there is sufficient terrain at the small areas to challenge a novice skier, whereas the intermediate and advanced skier, who generally skies much more and

TABLE V—*Continued*

	J =	Lake Eldora	Monarch	Mt. Werner	Wolf Creek	Purga-tory	Cooper	Hidden Valley
	N	0.0694	0.0085	0.0266	0.0214	0.0030	0.0178	0.0167
Denver	I	0.0475	0.0208	0.0458	0.0186	0.0040	0.0318	0.0122
	A	0.0416	0.0133	0.0333	0.0141	0.0045	0.0191	0.0105
	N	0.0859	0.0080	0.0263	0.0206	0.0029	0.0175	0.0221
Boulder	I	0.0590	0.0199	0.0457	0.0181	0.0039	0.0315	0.0163
	A	0.0516	0.0127	0.0331	0.0136	0.0045	0.0191	0.0141
Ft.	N	0.0755	0.0082	0.0275	0.0221	0.0032	0.0172	0.0282
Collins	I	0.0521	0.0203	0.0478	0.0194	0.0044	0.0311	0.0207
	A	0.0450	0.0128	0.0343	0.0145	0.0050	0.0186	0.0177
	N	0.0673	0.0086	0.0276	0.0226	0.0033	0.0174	0.0249
Greeley	I	0.0463	0.0212	0.0477	0.0198	0.0044	0.0313	0.0183
	A	0.0401	0.0134	0.0343	0.0148	0.0050	0.0187	0.0156
	N	0.0708	0.0081	0.0265	0.0204	0.0028	0.0182	0.0165
Golden	I	0.0482	0.0198	0.0454	0.0176	0.0037	0.0323	0.0120
	A	0.0422	0.0127	0.0330	0.0133	0.0043	0.0196	0.0104
Air Force	N							
Academy	I	0.0382	0.0306	0.0431	0.0231	0.0050	0.0315	0.0120
	A	0.0336	0.0197	0.0315	0.0175	0.0057	0.0191	0.0089
Colorado	N							
Springs	I	0.0351	0.0324	0.0435	0.0241	0.0051	0.0332	0.0095
	A	0.0307	0.0207	0.0316	0.0181	0.0058	0.0201	0.0082
	N	0.0462	0.0165	0.0244	0.0390	0.0051	0.0181	0.0119
Pueblo	I	0.0326	0.0417	0.0434	0.0351	0.0071	0.0333	0.0090
	A	0.0284	0.0266	0.0314	0.0263	0.0080	0.0201	0.0077
	N							
Alamosa	I	0.0194	0.0563	0.0415	0.0846	0.0131	0.0395	0.0051
	A							
	N	0.0274	0.0285	0.0232	0.0454	0.0101	0.0221	0.0066
Gunnison	I	0.0185	0.0687	0.0393	0.0389	0.0133	0.0388	0.0047
	A	0.0157	0.0429	0.0278	0.0285	0.0148	0.0229	0.0040
	N							
Durango	I	0.0166	0.0340	0.0324	0.1070	0.0572	0.0286	0.0051
	A	0.0134	0.0201	0.0218	0.0745	0.0604	0.0160	0.0041

much faster during a day of skiing, would quickly become bored with the small amounts of terrain. Ability levels in conjunction with the amount and types of terrain at the different areas affect the consumer's allocation of ski days.

5.B. *Price and Characteristic Elasticity Estimates*

Further insights can be gained into the nature of our estimated share equations and their underlying preference ordering by examining the share elasticities.

Site j's share elasticity with respect to a change in the price at site j, is:

$$E_{\hat{s}_j \hat{\gamma}_j} = -\hat{\sigma}\left(1 - \hat{s}_j\right). \tag{5.5}$$

These elasticity estimates are all negative, because $-\hat{\sigma}$ is negative. They vary between -1.24 and -1.98. Skiers are responsive to cost changes.

Site m's share elasticity with respect to a change in the price at site j is:

$$E_{\hat{s}_m \hat{\gamma}_j} = \hat{s}_j \hat{\sigma}. \tag{5.6}$$

These cross-elasticities are all positive by assumption. They fall between zero and 0.73. As the individual's predicted share for a given site increases, the more responsive his allocation among sites becomes to changes in that site's price.

We now examine site j's predicted share elasticities with respect to a change in the amount of a_{1j} at that site.

$$E_{\hat{s}_j a_{1j}} = E_{\hat{s}_j \gamma_j} E_{\hat{h}_j a_{1j}}. \tag{5.7}$$

$E_{\hat{h}_j a_{1j}}$ can be interpreted as a monotonic transformation of the proportionate amount by which the utility of a ski day increases when the amount of skiable terrain at the site increases by one percent. The elasticity (5.7) is equal to site j's predicted share elasticity with respect to a change in the amount of a_{6j} at the site.

$$E_{\hat{s}_j a_{1j}} \equiv E_{\hat{s}_j a_{6j}}. \tag{5.8}$$

This follows because $\Delta a_{1j} \equiv \Delta a_{6j}$ when a_{2j} and the other EPCs are held constant. These two elasticities measure how responsive a site's predicted share is to an increase in skiable terrain when the increase is completely in terms of terrain that is designed for individuals of lesser skiing ability. All of these characteristic elasticities are positive, except for advanced skiers at Vail (see Table VI). An individual's predicted share for a given site increases as the amount of terrain at that site on which he is capable of skiing increases. The more terrain he can ski on, the more variety he has, and the less bored he should become. This should increase his enjoyment of the area.

The individual's share elasticity for site j with respect to a change in the amount of a_{2j} is:

$$E_{\hat{s}_j a_{2j}} = E_{\hat{s}_j \gamma_j} E_{\hat{h}_j a_{2j}}. \tag{5.9}$$

These elasticities are most interesting. Table VII lists them for the residents of five cities. They are predominantly negative, but mixed in sign for advanced skiers. A negative characteristic elasticity of this type says that as the proportion of skiable terrain at an area designed specifically for your skiing ability increases, while holding total skiable terrain constant, your demand for skiing at that area decreases. These elasticity estimates are quite reasonable when we remember that skiing ability was measured in terms of the individual's capabilities rather than preferences. When an intermediate skier is defined as a skier who has the ability to ski on both novice and intermediate terrain, we should not be too surprised if he happens to enjoy the novice terrain relatively more. The intermediate terrain might be forcing him to the limits of his ability—a situation which every individual does not necessarily enjoy. If this is the case, one would expect his enjoyment of the site to in fact decrease when the amount of a_{2j} increases holding a_{1j} constant. This argument might also explain the fact that the elasticities for advanced skiers at Aspen, Vail, and a few of the other sites are positive. The advanced category is open ended and will therefore include people who enjoy skiing the advanced terrain and others who do not. For example,

TABLE VI

$E_{\hat{s}_j a_{1j}}$

	$J =$	Aspen	Vail	A-Basin	Breckenridge	Loveland	Winter Park	Broadmoor	Crested Butte
	N	2.271	4.339	1.686	1.606	1.610	1.246	0.2103	1.835
Denver	I	2.441	5.077	1.257	1.349	1.251	0.9531	0.1336	0.7292
	A	0.0103	−1.689	0.8303	0.9587	0.5271	0.4849	0.1186	0.7412
Ft.	N	2.234	4.330	1.696	1.610	1.626	1.261	0.2122	1.833
Collins	I	2.405	5.064	1.265	1.353	1.266	0.9625	0.1342	0.7286
	A	0.0101	−1.686	0.8365	0.9625	0.5328	0.4889	0.1171	0.7409
	N	2.171	4.380	1.729	1.611	1.676	1.315	0.1956	1.804
Pueblo	I	2.335	5.097	1.294	1.351	1.312	0.9987	0.1284	0.7172
	A	0.0096	−1.702	0.8528	0.9608	0.5472	0.5012	0.1144	0.7337
	N	2.081	4.268	1.732	1.614	1.698	1.362	0.2138	1.673
Gunnison	I	2.282	5.023	1.302	1.360	1.340	1.039	0.1348	0.6714
	A	0.0093	−1.667	0.8587	0.9686	0.5575	0.5162	0.1174	0.7057
	N								
Durango	I	2.087	5.139	1.320	1.382	1.362	1.050	1.358	0.7063
	A	0.0083	−1.754	0.8716	0.9871	0.5664	0.5215	0.1180	0.7284

	$J =$	Lake Eldora	Monarch	Mt. Werner	Wolf Creek	Purgatory	Cooper	Hidden Valley
	N	0.6839	1.718	2.028	0.7106	2.183	2.856	0.8440
Denver	I	1.026	1.358	1.557	0.7922	1.457	1.551	0.9136
	A	0.4333	0.8715	0.6701	0.7523	1.342	0.6233	1.215
Ft.	N	0.6794	1.718	2.016	0.7100	2.182	2.857	0.8341
Collins	I	1.021	1.359	1.553	0.7916	1.456	1.552	0.9057
	A	0.4318	0.8719	0.6694	0.7519	1.342	0.6237	1.207
	N	0.7009	1.704	2.023	0.6978	2.178	2.855	0.8481
Pueblo	I	1.042	1.329	1.560	0.7789	1.452	1.549	0.9166
	A	0.4393	0.8598	0.6714	0.7429	1.337	0.6228	1.219
	N	0.7147	1.683	2.025	0.6931	2.167	2.843	0.8526
Gunnison	I	1.057	1.292	1.567	0.7759	1.443	1.540	0.9205
	A	0.4450	0.8453	0.6738	0.7412	1.328	0.6210	1.223
	N							
Durango	I	1.059	1.340	1.578	0.7209	1.379	1.556	0.9202
	A	0.4461	0.8654	0.6783	0.7061	1.267	0.6254	1.223

the advanced skiers at Vail must gain their most enjoyment by skiing on advanced terrain.

Given the fact that the individual generally reacts adversely when the proportion of the terrain at a site designed specifically for his level of skiing ability increases (a_{1j} constant), one wonders what happens to site j's predicted share when the amount of a_{2j} and a_{1j} increase in equal amounts. The elasticity measuring this response equals the sum of the characteristic elasticities (5.7) and (5.9).

$$E_{\hat{s}_j a_{2j}} \mid \Delta a_{2j} = \Delta a_{1j} = E_{\hat{s}_j a_{1j}} + E_{\hat{s}_j a_{2j}}. \tag{5.10}$$

Examination of Tables VI and VII show that these are predominantly positive except for Aspen and Vail. These characteristic elasticities generally lead to the conclusion that a site's predicted share increases when a_{6j} or a_{2j} increases, but that novice and intermediate skiers respond more positively to an increase in a_{6j} than an

TABLE VII

$E_{\hat{s}_j a_{2j}}$

	$J =$	Aspen	Vail	A-Basin	Brecken-ridge	Love-land	Winter Park	Broad-moor	Crested Butte
	N	−2.415	−4.564	−1.012	−0.9152	−1.042	−0.8953	−0.3824	−1.294
Denver	I	−2.680	−6.335	−0.5444	−0.6353	−0.6515	−0.4440	−0.1169	−0.0251
	A	0.0748	1.097	−0.1230	−0.2334	0.0645	−0.6210	−0.1168	−0.0347
Ft.	N	−2.375	−4.555	−1.617	−0.9177	−1.052	−0.9058	−0.3838	−1.293
Collins	I	−2.639	−6.318	−0.5479	−0.6372	−0.6591	−0.4484	−0.1174	−0.0251
	A	0.6732	1.095	−0.1239	−0.2343	0.0652	0.0212	−0.1171	−0.0347
	N	−2.308	−4.607	−1.038	−0.9183	−1.085	−0.9446	−0.3556	−1.272
Pueblo	I	−2.563	−6.359	−0.5606	−0.6364	−0.6829	−0.4625	−0.1123	−0.0247
	A	0.0696	1.106	−0.1263	−0.2339	0.0670	0.0217	−0.1144	−0.0343
	N	−2.212	−4.489	−1.039	−0.9200	−1.099	−0.9785	−0.3887	−1.180
Gunnison	I	−2.505	−6.267	−0.5640	−0.6406	−0.6978	−0.4841	−0.1180	−0.0231
	A	0.0676	1.083	−0.1272	−0.2358	0.0682	0.0224	−0.1174	−0.0330
	N								
Durango	I	−2.291	−6.412	−0.5718	−0.6511	−0.7091	−0.4890	−0.1189	−0.0243
	A	0.0600	1.139	−0.1291	−0.2403	0.0693	0.0226	−0.1180	−0.0341

	$J =$	Lake Eldora	Monarch	Mt. Werner	Wolf Creek	Purga-tory	Cooper	Hidden Valley
	N	−0.4749	−0.3435	−0.9991	−0.0789	−0.8237	−1.632	−0.5137
Denver	I	−0.7327	−0.3134	−0.6671	−0.0660	−0.2869	−0.5171	−0.4299
	A	−0.0228	0.1226	0.1197	0.0	−0.2833	0.2032	−0.7125
Ft.	N	−0.4718	−0.3436	−0.9981	−0.0788	−0.8235	−1.633	−0.5077
Collins	I	−0.7292	−0.3136	−0.6657	−0.0659	−0.2868	−0.5175	−0.4262
	A	−0.0227	0.1226	−1195	0.0	−0.2832	0.2034	−0.7073
	N	−0.4868	−0.3407	−1.001	−0.0775	−0.8220	−1.631	−0.5162
Pueblo	I	−0.7422	−0.3067	−0.6687	−0.0649	−0.2860	−0.5163	−0.4313
	A	−0.6231	0.1189	0.1203	0.0	−0.2824	0.2031	−0.7145
	N	−0.4963	−0.3366	−1.003	−0.0770	−0.8179	−1.625	−0.5190
Gunnison	I	−0.7551	−0.2981	−0.6716	−0.0646	−0.2843	−0.5134	−0.4332
	A	−0.0234	0.1189	0.1203	0.0	−0.2804	0.2025	−0.7172
	N							
Durango	I	−0.7565	−0.3092	−0.6764	−0.0600	−0.2716	−0.5188	−0.4330
	A	−0.0234	0.1217	0.1211	0.0	−0.2675	0.2039	−0.7171

increase in a_{2j}. For advanced skiers their relative preferences for increases in a_{2j} and a_{6j} are mixed.

The individual's share elasticities for site j with respect to a_{3j} and a_{4j} are:

$$E_{\hat{s}_j a_{nj}} = -E_{\hat{s}_j \gamma_j} E_{\hat{h}_j a_{nj}}, \qquad n = 3, 4. \tag{5.11}$$

The elasticities w.r.t. a_{3j}(VTF_j) are predominantly positive (see Table VIII for examples). This is as expected. Normally as VTF increases, everything else constant, lift lines decrease and more skiing becomes possible. However, if VTF is very high in proportion to skiable terrain, the slopes will become quite crowded and one would then expect that increasing VTF will make skiing less enjoyable. This explains the negative elasticities for Wolf Creek and some ability categories at other sites (novices

TABLE VIII

$E_{\$_j a_{3j}}$

	$J =$	Aspen	Vail	A-Basin	Brecken-ridge	Love-land	Winter Park	Broad-moor	Crested Butte
	N	−0.0277	−0.4005	0.4838	0.3853	0.3077	0.5538	2.944	1.622
Denver	I	0.1153	−0.1281	0.3282	0.2698	0.2322	0.4381	2.605	1.220
	A	0.3533	0.3796	0.3260	0.2712	0.3496	0.5165	2.604	1.135
	N	−0.0253	−0.3939	0.4970	0.3874	0.3246	0.6053	2.993	1.479
Gunnison	I	0.1078	−0.1267	0.3400	0.2720	0.2487	0.4776	2.629	1.123
	A	0.3192	−0.3747	0.3371	0.2740	0.3698	0.5499	2.617	1.081
					$E_{\$_j a_{4j}}$				
	N	−3.628	−0.9429	−1.114	−3.051	−2.190	−2.280	−2.141	−0.6567
Denver	I	−1.538	1.875	−0.0400	−0.9921	−0.6128	−1.131	−1.887	−0.5783
	A	−0.8209	0.8250	0.1661	−0.4604	−0.5863	−1.049	−1.853	−0.3359
	N	−3.324	−0.9274	−1.145	−3.067	−2.309	−2.492	−2.176	−0.5987
Gunnison	I	−1.438	1.855	−0.0414	−1.000	−0.6563	−1.233	−1.904	−0.5325
	A	−0.7417	0.8144	0.1717	−4.652	−0.6201	−1.117	−1.862	−0.3198

$E_{\$_j a_{3j}}$

	$J =$	Lake Eldora	Monarch	Mt. Werner	Wolf Creek	Purga-tory	Cooper	Hidden Valley
	N	2.489	0.0381	−0.0999	−0.4145	0.9061	1.412	4.293
Denver	I	1.949	0.0298	0.0	−0.2641	0.5518	0.7182	3.171
	A	1.631	0.0817	0.1755	−0.1991	0.3877	0.7994	2.473
	N	2.601	0.0374	−0.1003	−0.4043	0.8996	1.406	4.337
Gunnison	I	2.009	0.0283	0.0	−0.2586	0.5466	0.7130	3.195
	A	1.675	0.0792	0.1765	−0.1962	0.3838	0.7965	2.489
					$E_{\$_j a_{4j}}$			
	N	−2.678	13.13	−0.9991	7.816	0.7413	3.169	−4.440
Denver	I	−1.363	6.194	0.3891	5.691	0.8387	2.126	−2.795
	A	−1.186	5.447	0.0957	4.923	1.163	1.734	−1.425
	N	−2.799	12.86	−1.003	7.625	0.7361	3.156	−4.486
Gunnison	I	−1.404	5.891	0.3918	5.573	0.8309	2.111	−2.816
	A	−1.218	5.283	0.0962	4.851	1.151	1.728	−1.434

at Mt. Werner, novices and intermediates at Vail and novices at Aspen). The ratios VTF_j/a_{1j} are very high for these groups.

The elasticities w.r.t. a_{4j} (average annual snowfall) are mixed in sign. They vary from −4.4 to 13.1 (see Table VIII for examples), and tend to increase as a_{4j} increases. More snow, at those sites known for their large snowfall (e.g., Monarch and Wolf Creek), has a strong positive effect on their shares. Novices react more than intermediates to changes in a_{4j}, and intermediates react more than advanced skiers. Snowfall influences the skiing experience in a number of ways. Increased snow, assuming proper grooming, makes the snow more enjoyable to ski on, but visibility is detrimentally affected while the snow is actually falling. The magnitude of these opposing effects for different groups at the different sites might explain the variations in the elasticities.

The cross-elasticities with respect to the four EPCs ($a_{1j}, a_{2j}, a_{3j}, a_{4j}$) are:

$$E_{\hat{s}_m a_{nj}} = -E_{\hat{s}_m \gamma_j} E_{\hat{h}_j a_{nj}}, \qquad n = 1, 2, 3, 4. \tag{5.12}$$

The ski area business is extremely competitive, which makes the operators quite interested in their market shares. The individual areas could use the estimated elasticities to increase their ticket sales and market shares. For example, if A-Basin had increased its 1967/1968 intermediate acreage by one percent, by converting advanced terrain into intermediate terrain, the model predicts that the proportion of the advanced skiers' trips to A-Basin would have increased by approximately 0.12%, the proportion of intermediate skiers' trips to A-Basin would have increased by approximately 0.71%, and the proportion of novice skiers' trips would have remained unchanged. This suggests that A-Basin could have sold more tickets simply by grooming out some moguls. Whereas if Loveland had done the same thing, the proportion of advanced skiers' trips to Loveland would have decreased by approximately 0.06%, and the proportion of intermediate skiers' trips would have increased by approximately 0.60%. The optimality of such a move would then depend on how popular the area is with intermediates relative to advanced skiers. Maybe in winter the conversion would not be advisable, but if springtime brings out relatively more intermediates, then maybe Loveland should have fewer moguls in the spring. This phenomenon seems to occur in the later part of the season at many areas. The effect of changing lift ticket prices or other physical characteristics could also be calculated.

The model can be used to calculate predicted shares for a proposed site. One just needs to know: the site's proposed location, average annual snowfall, types and amount of terrain, VTF, and anticipated ticket price.

The model can be aggregated across individuals to provide ticket sale estimates. This just requires additional data on the number and types of skiers residing in each city. The model assumes that total ski days are predetermined; this is a deficiency, but not one that makes the model void of policy implications. Making total ski days an endogenous variable would require a much more complex model for which the data are not available.

6. SUMMARY AND CONCLUSIONS

The purpose of this research was four-fold. First, I obtained a system of share equations for site-specific recreational activities which are consistent with an underlying theory of constrained utility maximizing behavior. Second, I incorporated the important physical characteristics of the recreational sites directly into the utility function in such a way that the individual's production technology (in this case skiing ability) would limit the individual's ability to utilize the characteristics. Inclusion of the EPCs resulted in identical share equations, a useful result. Third, I specified a density function for the shares that is consistent with the shares' inherent properties. Finally, I estimated this model and confirmed my basic hypothesis that both prices and the EPCs play an important explanatory role in the individual's allocation of ski days amongst sites.

Most of the previous work on recreational demand has utilized the travel-cost technique (pioneered by Clawson [8]), a technique which recognizes the strong statistical relationship between a site's predicted share and the distance from that site to the individual's residence, but lacks a strong foundation in basic consumer theory. My model recognizes and confirms the hypothesis that travel costs are important and also gives it a strong theoretical foundation.

A large proportion of the variation in the data was explained by the model. The elasticity estimates are not unreasonable, and offer practical insights into the behavior of the skier. The nonquantified operating postulates of the managements of most ski areas are confirmed and quantified.

APPENDIX: HOW THE CONDITIONAL MULTINOMIAL (POLYCHOTOMOUS) LOGIT MODEL CAN BE USED TO MODEL THE CONSTRAINED UTILITY MAXIMIZING BEHAVIOR OF INDIVIDUALS WHO ARE TRYING TO DECIDE WHERE TO SKI[18]

The conditional multinomial logit model (hereafter referred to as the logit model) was designed to explain population choice on the basis of individual decision rules, where the individuals take explicit account of the fact that there is only a small number of alternatives from which to choose. This suggests that it might be applicable to the problem of determining where the individuals in a population will ski. The skiers analyzed have only 15 alternative areas from which to choose. A visit to one precludes the possibility of visiting another on the same day, so the choice set is lumpy. My intent is to show that logit analysis can be used to model the skiers' behavior. It turns out that my model explains more of the variation in the data than the logit model.

The individual, on each day he goes skiing, is able to choose from amongst 15 ski areas. Let B describe this set of alternatives:

$$B = \{X^1, X^2, \ldots, X^{15}\},$$

where X^j is a vector of the characteristics of a day trip to site j. On a given day the individual will choose to visit the site that provides the greatest amount of utility. The utility he receives from one day of skiing at site j is:[19]

$$U(X^j, C) = V(X^j, C) + \eta(X^j, C), \tag{A.1}$$

where

$V(X^j, C)$ is identical for all individuals with the vector of socioeconomic characteristics C.

$\eta(X^j, C)$ varies across those individuals.

[18] For an extensive survey of conditional multinomial logit models, see McFadden [19] and/or Domencich and McFadden [14].

[19] It should be noted that the utility the individual receives from a day trip to site j does not depend on where the individual skied or plans to ski on other days. There is constant returns to scale in the production of utility from one-day trips to site j, i.e., diminishing marginal utility associated with multiple visits to a site is disallowed by assumption. Ski days are strongly separable (additive) in the individual's seasonal utility function. This strong separability across days in conjunction with the fact that only one site can be visited per day implies that the utility function is also effectively additive across sites (the utility received from site j is independent of the characteristics of the other sites).

The probability that an individual randomly drawn from the population will choose to visit site j on a given day is therefore

$$P_j = \text{Prob}\left[U(X^j, C) > U(X^k, C) \text{ for } k \neq j,\ k = 1, 2, \ldots, 15\right],$$
$$= \text{Prob}\left[\eta(X^k, C) - \eta(X^j, C) < V(X^j, C) - V(X^k, C) \quad \text{for } k \neq j,\ k = 1, 2, \ldots, 15\right]. \tag{A.2}$$

Logit analysis specifically assumes that the random variable $\eta_i(X^j, C)$—where i refers to individual i—has a Weibull distribution.

$$\text{Prob}[\eta_i \leq \eta] = e^{-e^{-(\eta+\alpha)}}, \tag{A.3}$$

in which case the probability that individual i with socioeconomic characteristics C_i will visit site j on a given day is:[20]

$$P_{ji} = \text{Prob}\left[X_i^j \mid C_i\right] = e^{V_i(X_i^j, C_i)} \Big/ \sum_{k=1}^{15} e^{V_i(X_i^k, C_i)}. \tag{A.4}$$

Logit analysis normally assumes that the nonstochastic component of the utility function is a function of the EPCs of the alternative chosen. The EPCs in my model are $a_{1j} - a_{4j}$, and γ_j (the cost of a one-day trip to site j). I therefore assume that:[21]

$$V(X^j, C) = \beta_0 + \beta_1\gamma_1 + \beta_2 a_{1j} + \beta_3 a_{2j} + \beta_4 a_{3j} + \beta_5 a_{4j} \tag{A.5}$$

It then follows that:

$$P_{ji} = \text{Prob}\left[X_i^j \mid C_i\right] = 1 \Big/ \sum_{k=1}^{15} e\left[\beta_1(\gamma_k - \gamma_j) + \beta_2(a_{1k} - a_{1j}) + \beta_3(a_{2k} - a_{2j}) + \beta_4(a_{3k} - a_{3j}) + \beta_5(a_{4k} - a_{4j})\right]. \tag{A.6}$$

If an individual's choice of ski trips during the ski season are statistically independent, then the probability of observing a given vector of shares $(s_1, s_2, \ldots, s_{15})$ for an individual randomly chosen from the population is:

$$f(s_{1_i}, s_{2_i}, \ldots, s_{15_i}) = \frac{T_i!}{\prod_{j=1}^{15} y_{ji}!} \prod_{j=1}^{15} (P_{ji})^{y_{ji}}, \tag{A.7}$$

[20] For details see Domencich and McFadden [14, pp. 61–69].

[21] It is of course possible to generalize this function somewhat without abandoning its linear form. For example, I could have used a linear 2nd-order approximation to any function in five variables. This would substantially increase the number of variables.

TABLE IX

Maximum Likelihood Estimates for the Logit Model

l*	B_1	B_2	B_3	B_4	B_5
−3528.001	−0.117902	0.000208	−0.000314	0.000152	0.003610

where

y_{ji} is the number of units of ski activity j produced per season by individual i, where one unit of y_j is one day of skiing at site j.

$$T_i \equiv \sum_{k=1}^{15} y_{ki}, \qquad s_{ki} \equiv y_{ki}/T_i.$$

If the choice of sites by one individual is completely independent of any other individual's choice, the log of the likelihood function for a sample of 163 skiers is:

$$\begin{aligned} l &= \sum_{i=1}^{163} \sum_{j=1}^{15} y_{ji} \log(P_{ji}) \\ &= -\sum_{i=1}^{163} \sum_{j=1}^{15} y_{ji} \log\Bigg[\sum_{k=1}^{15} e\{\beta_1(\gamma_k - \gamma_j) + \beta_2(a_{1k} - a_{1j}) + \beta_3(a_{2k} - a_{2j}) \\ &\qquad + \beta_4(a_{3k} - a_{3j}) + \beta_5(a_{4k} - a_{4j})\}\Bigg]. \end{aligned}$$

The maximum likelihood estimates of the B parameters were obtained using a Newton-type search algorithm. McFadden [19, pp. 119–120] and others, have shown that, under very general conditions, the maximum likelihood estimates are consistent, asymptotically efficient, and asymptotically normally distributed. The modified R^2 and examination of actual and predicted shares give us an indication of the model's goodness of fit vis-a-vis my model. Examination of these statistics indicates that my model predicts the skiers choice of sites better than the logit model. The predicted shares (the probability that a given individual will visit a given site on a given day), are reported in Table XI. The logit model has some advantages but I think that overall my model is better than the logit model in explaining skier behavior.

TABLE X

Actual and Predicted Shares—The Logit Model

	Aspen	Vail	A-Basin	Breckenridge	Loveland	Winter Park	Broadmoor	Crested Butte	Lake Eldora	Monarch	Mt. Werner	Wolf Creek	Purgatory	Cooper	Hidden Valley
Actual share	0.20	0.16	0.21	0.07	0.09	0.16	0.01	0.01	0.06	0.03	0.05	0.01	0.01	0.01	0.01
Share predicted by the model	0.22	0.15	0.07	0.08	0.10	0.09	0.03	0.01	0.05	0.04	0.04	0.03	0.01	0.04	0.05
% Bias	+2%	−1%	−5%	+1%	+1%	−7%	+2%	0	−1%	+1%	−1%	+2%	0	+3%	+4%

TABLE XI

$\hat{P}_{vi}$, The Estimated Probabilities

	J =	Aspen	Vail	A-Basin	Brecken-ridge	Love-land	Winter Park	Broad-moor	Crested Butte
	N	0.1931	0.1111	0.0856	0.0895	0.1156	0.1034	0.0281	0.0119
Denver	I	0.1966	0.1094	0.0857	0.0889	0.1158	0.1044	0.0278	0.0121
	A	0.2337	0.2024	0.0698	0.0720	0.0968	0.0858	0.0219	0.0096
	N	0.1892	0.1089	0.0839	0.0877	0.1132	0.1013	0.0239	0.0110
Boulder	I	0.1927	0.1072	0.0840	0.0872	0.1135	0.1024	0.0237	0.0112
	A	0.2300	0.1992	0.0687	0.0708	0.0953	0.0844	0.0187	0.0088
Ft.	N	0.1848	0.1063	0.0819	0.0856	0.1106	0.0989	0.0246	0.0105
Collins	I	0.1883	0.1047	0.0821	0.0852	0.1109	0.1000	0.0244	0.0106
	A	0.2258	0.1955	0.0674	0.0696	0.0935	0.0829	0.0194	0.0085
	N	0.1874	0.1074	0.0831	0.0868	0.1121	0.1003	0.0268	0.0114
Greeley	I	0.1909	0.1061	0.0832	0.0863	0.1124	0.1014	0.0265	0.0116
	A	0.2282	0.1977	0.0682	0.0703	0.0945	0.0838	0.0210	0.0092
	N	0.1959	0.1128	0.0868	0.0908	0.1173	0.1049	0.0223	0.0114
Golden	I	0.1995	0.1109	0.0869	0.0902	0.1175	0.1060	0.0221	0.0116
	A	0.2363	0.2047	0.0706	0.0728	0.0979	0.0867	0.0173	0.0091
Air Force	N								
Academy	I	0.2173	0.0948	0.0705	0.0870	0.0952	0.0859	0.0616	0.0218
	A	0.2607	0.1770	0.0580	0.0711	0.0804	0.0712	0.0489	0.0174
Colorado	N								
Springs	I	0.2300	0.1003	0.0627	0.0921	0.0847	0.0764	0.0652	0.0230
	A	0.2731	0.1855	0.0510	0.0745	0.0708	0.0627	0.0513	0.0182
	N	0.2206	0.0913	0.0561	0.0830	0.0757	0.0677	0.0632	0.0290
Pueblo	I	0.2247	0.0898	0.0562	0.0825	0.0759	0.0684	0.0626	0.0295
	A	0.2707	0.1685	0.0464	0.0677	0.0643	0.0570	0.0499	0.0236
	N								
Alamosa	I	0.2218	0.0993	0.0466	0.0673	0.0529	0.0368	0.0203	0.0335
	A								
	N	0.2481	0.0999	0.0457	0.0671	0.0519	0.0358	0.0166	0.0927
Gunnison	I	0.2524	0.0983	0.0457	0.0666	0.0519	0.0361	0.0165	0.0940
	A	0.2987	0.1812	0.0371	0.0537	0.0432	0.0295	0.0129	0.0739
	N								
Durango	I	0.2839	0.0476	0.0221	0.0322	0.0251	0.0175	0.0058	0.0360
	A	0.3524	0.0920	0.0188	0.0273	0.0219	0.0150	0.0047	0.0297

TABLE XI—*Continued*

	J =	Lake Eldora	Monarch	Mt. Werner	Wolf Creek	Purga-tory	Cooper	Hidden Valley
	N	0.0574	0.0369	0.0427	0.0292	0.0071	0.0452	0.0432
Denver	I	0.0566	0.0364	0.0424	0.0288	0.0071	0.0453	0.0427
	A	0.0454	0.0292	0.0349	0.0228	0.0056	0.0368	0.0336
	N	0.0699	0.0340	0.0419	0.0269	0.0066	0.0443	0.0574
Boulder	I	0.0689	0.0336	0.0416	0.0266	0.0065	0.0444	0.0568
	A	0.0555	0.0270	0.0344	0.0211	0.0052	0.0362	0.0448
Ft.	N	0.0682	0.0324	0.0409	0.0263	0.0062	0.0433	0.0794
Collins	I	0.0673	0.0319	0.0406	0.0260	0.0062	0.0434	0.0785
	A	0.0545	0.0258	0.0338	0.0207	0.0049	0.0355	0.0623
	N	0.0597	0.0352	0.0415	0.0278	0.0068	0.0439	0.0695
Greeley	I	0.0589	0.0347	0.0412	0.0275	0.0067	0.0440	0.0687
	A	0.0475	0.0280	0.0341	0.0219	0.0053	0.0360	0.0543
	N	0.0567	0.0352	0.0434	0.0279	0.0068	0.0459	0.0420

TABLE XI—*Continued*

	$J =$	Lake Eldora	Monarch	Mt. Werner	Wolf Creek	Purga-tory	Cooper	Hidden Valley
Golden	I	0.0559	0.0347	0.0430	0.0275	0.0067	0.0460	0.0415
	A	0.0447	0.0277	0.0353	0.0216	0.0053	0.0372	0.0325
Air Force	N							
Academy	I	0.0405	0.0595	0.0349	0.0369	0.0090	0.0440	0.0352
	A	0.0377	0.0481	0.0290	0.0294	0.0072	0.0360	0.0279
Colorado	N							
Springs	I	0.0414	0.0629	0.0350	0.0391	0.0096	0.0465	0.0313
	A	0.0332	0.0504	0.0288	0.0308	0.0075	0.0377	0.0246
	N	0.0376	0.0867	0.0316	0.0674	0.0164	0.0453	0.0283
Pueblo	I	0.0371	0.0856	0.0314	0.0666	0.0163	0.0455	0.0280
	A	0.0302	0.0695	0.0262	0.0533	0.0130	0.0374	0.0223
	N							
Alamosa	I	0.0142	0.1068	0.0247	0.1741	0.0426	0.0502	0.0091
	A							
	N	0.0141	0.1288	0.0247	0.0707	0.0456	0.0497	0.0087
Gunnison	I	0.0139	0.1269	0.0245	0.0697	0.0452	0.0497	0.0086
	A	0.0111	0.1013	0.0201	0.0549	0.0355	0.0402	0.0067
	N							
Durango	I	0.0067	0.0469	0.0086	0.2151	0.2228	0.0241	0.0056
	A	0.0056	0.0393	0.0074	0.1775	0.1835	0.0204	0.0046

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