

TWO NESTED CONSTANT-ELASTICITY-OF-SUBSTITUTION MODELS OF RECREATIONAL PARTICIPATION AND SITE CHOICE: AN “ALTERNATIVES” MODEL AND AN “EXPENDITURES” MODEL

EDWARD R. MOREY, WILLIAM S. BREFFLE, AND PAMELA A. GREENE

Two demand models of recreational participation and site choice are developed: an alternatives model and an expenditures model. Both assume maximization of utility over the year, so allow for diminishing marginal utility. They do not impose the restrictive assumption that where one goes on a trip is independent of where one plans to go on other occasions. Estimation is with a nested constant-elasticity-of-substitution preference ordering: it is relatively easy to estimate because of global regularity, it allows sites to be complements, and it has the potential to be locally flexible. The application is to Atlantic salmon fishing.

Key words: atlantic salmon fishing, neoclassical recreation demand, nested CES.

Two complete demand models of recreational participation and site choice are developed and estimated: an alternatives model and an expenditures model. Both models are rooted in neoclassical demand theory. Both assume the individual maximizes utility over the whole year and allow for diminishing or increasing marginal utility associated with trips to a particular site and recreation in general. They also allow sites or groups of sites to be complements. In contrast, the repeated discrete-choice random utility model (RUM) of participation and site choice assumes all alternatives are substitutes.¹

Edward Morey is professor in the Department of Economics, University of Colorado at Boulder. William Breffle is senior associate at Stratus Consulting, Inc., Boulder, Colorado. Pamela Greene is principal analyst at the Congressional Budget Office, Washington D.C.

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¹ Repeated discrete-choice random utility models evolved from discrete-choice models of site choice by dividing the year into a finite number of independent choice-occasions, and later added nonparticipation as one of the alternatives available during each choice-occasion. They typically assume choices are statis-

incorporating complementarity is important; if one type of recreation is improved, participants might become more avid in all types of recreation. In our choice set, one region of fishing sites has much better catch rates than the other region, making the fishing experiences quite different. If fishing quality were improved in one of the regions, it would not be unreasonable for anglers to become more avid about fishing in general, and thus to fish more in both regions. Complementarity has important implications for estimated trip patterns and welfare estimates.

The alternatives model can be viewed as a generalization of the trip-share model of site choice, generalized to explain both participation and site choice. See Morey (1981, 1984, and 1985) and Morey and Shaw. The trip-share model explains only the choice

tically independent across choice occasions. Random parameter models admit correlation across choice-occasions, but preferences are not defined over the year and marginal utility for an alternative is assumed to be constant. Note that there are other models of participation and site choice, such as *blended* models: a site-choice model is estimated and used to generate an estimate of the expected net benefits of taking a trip, and then the total number of trips is estimated as a function of that index. Blended models typically define preferences over the entire year assuming that trips are separable from other commodities. Thus, they are consistent with diminishing marginal utility for total trips, but are not consistent with a demand for variety across sites, except for Shonkwiler and Shaw. The blended models do not admit complements.

of sites given the decision to participate, so has been abandoned in favor of models such as repeated RUMs that explain both participation and site choice.

Estimating parameters of the utility function using expenditures (either levels or shares) has a long tradition in empirical demand theory, but not in recreational demand. Consider, for example, the *linear expenditures system* and the *quadratic expenditures system*. For examples of models where either expenditures or expenditures shares are the dependent variable, see Deaton and Muellbauer; Howe, Pollak, and Wales; Pollak and Wales (1987); Wales and Woodland; and Woodland.

Estimation of the alternatives model and expenditures model is with a nested constant-elasticity-of-substitution (NCES) preference ordering. To our knowledge, this is the first application of the NCES to recreational demand. The NCES has many desirable properties: it is relatively easy to estimate because it is globally regular, it allows sites to be complements, and it has the potential to be locally flexible. Note, however, that estimation of the two proposed models is possible with any well-behaved preference ordering.

Both models are estimated using a sample of Maine residents who held Atlantic salmon fishing licences. For comparison, a repeated RUM and a constant-elasticity-of-substitution (CES) alternatives model are also estimated. The predicted number of trips to each site under current fishing conditions are compared across the models. In addition, expected compensating variations ($E(CV)$ s) and changes in the number of trips are compared across the models for three policy-relevant changes in fishing conditions at the Penobscot River, a popular Maine fishing site.

Modeling Participation

Estimation of the participation decision is accomplished, as is typical, with no data on the price or quantity consumed of other commodities or activities (i.e., nonparticipation in the recreational activity of interest). To circumvent this lack of data, the expenditures model assumes dollars not spent on recreational activities are spent on a numeraire for which everyone pays the same price, \$1. In this case, units consumed of the numeraire equal expenditures on everything else, which

is known via budget exhaustion. If two individuals consume the same trip vector, but one has a higher income, the individual with the higher income consumes more units of the numeraire.

In contrast, the alternatives model circumvents the lack of price and quantity data for other activities by assuming the year consists of a finite number of occasions to participate in recreation, and that nonparticipation is an activity with a cost just like the cost of a trip. While not literally true, assuming a finite number of occasions to recreate is a proven modeling technique and reflects the time-intensive nature of fishing. In this case, the number of times something else is chosen equals the number of occasions minus the number of trips, and the price of nonparticipation is the individual's expenditures on everything else divided by the number of times the individual did not choose recreation. The cost of doing something else varies across individuals because different people choose different goods and activities when they are not on a trip to a recreational site. If two individuals consume the same trip vector but one has a higher income, the individual with the higher income participates in more expensive activities when he is doing something else. There is no numeraire per se.

Compare these two models with a typical repeated discrete-choice RUM of participation and site choice (hereafter a repeated RUM), which must also circumvent the lack of data on the price and quantity of nonparticipation. That model assumes the following: the year consists of a finite number of occasions to recreate, income for the year is pre-allocated among the occasions, the cost of staying home is zero, and per-occasion income not spent taking a trip is spent on a numeraire with a price of \$1. In addition, preferences are defined over the occasion, not the year, so during each occasion the individual maximizes utility for the occasion independent of what he or she chose on other occasions, so each occasion is a *choice* occasion. This is in contrast to the alternatives model, which like the repeated RUM model assumes the year consists of a finite number of occasions to recreate, but assumes preferences are defined over the year, so choices are made to maximize utility for the year. The expenditures model also assumes preferences are defined over the year.

The NCES Preference Ordering

The NCES has a history with respect to estimating the elasticities of substitution between inputs to production, such as capital and energy, when intermediate goods are produced and further refined in the production process. Sato was the first to develop a NCES model by aggregating intermediate inputs into an index nested within a broader CES functional form. Khan estimates a two-stage NCES manufacturing function for Pakistan with three factors: capital, energy, and labor, where the first two inputs are combined at the intermediate stage to produce a working machine. Because the NCES allows complementarity, it is at the heart of the debate on whether capital and energy are substitutes or complements. See also Prywes for a three-level NCES production function, and Pollak and Wales (1987) for a two-level NCES production function, for U.S. and Dutch manufacturing, respectively. Brown and Heien use a two-level NCES to represent the structure of consumer preferences for detailed categories of food, the first (and apparently only) empirical consumer demand application of the NCES.

The Multinomial Distribution

The alternatives model assumes that each individual's observed trip vector (number of trips to each site) is generated by random draws from a multinomial distribution with probabilities that vary systematically across individuals. The expenditures model assumes that each individual's observed expenditures vector (expenditures by site) is generated by random draws from a multinomial distribution. The multinomial assumption, while universal in repeated RUMs, is not common for other demand systems, but does have a history in recreational demand. Morey (1981) first proposed the multinomial for observed trip vectors generated by maximizing utility from recreation over the entire year, given some recreation budget.

In the alternatives model, the multinomial probabilities are assumed to be the alternatives share functions derived from the Marshallian demands, where the alternatives share for a site is the proportion of occasions that trips are taken to that site. In the expenditures model, the multinomial probabilities

are the expenditures shares, where the expenditures share for a site is the proportion of income spent on trips to that site. Therefore, the multinomial probabilities are functions of demand parameters and observed individual and site characteristics.

The multinomial assumption, unlike the common normality assumption, is consistent with the inherent properties of trips and expenditures: trips and expenditures cannot be negative, total expenditures must just exhaust the budget, and the total sum of activities selected must equal the number of occasions. Also note that the multinomial distribution is consistent with the preponderance of corner solutions that are observed in recreational data sets.

The expenditures model brings to the forefront an important, but often overlooked, issue associated with assuming units of the commodity (time or money) are random draws from a multinomial distribution. The intent of the alternatives model and the repeated RUM is to explain the allocation of time; in the expenditures model it is to explain how money is allocated. An important issue is the appropriate units in which to measure money and time. For example, is it more appropriate to assume that single dollar units, \$10 units, or \$100 units are distributed multinomially? For models explaining the allocation of time, should days, trips, hours, or minutes be used? The assumption has consequences, and the appropriate unit depends on how individuals think about their time and money.

In explaining the allocation of time across sites in site-choice only models, modelers typically assume the unit of time that is multinomially distributed is days or trips. Using hours instead would greatly increase the number of units.² Alternatively, in discrete-choice models of participation and site choice, it is often assumed that the year consists of the same finite number of choice occasions for each individual. For example, we chose 100. The number chosen is typically a few hundred or less because nobody takes more than a few hundred trips per year.

Units are important because, when the magnitude of the units is decreased (e.g., converted from days to hours), the total number

² While days or trips might seem to be the natural units, is it because they are the appropriate units, or because it is what we have always assumed? If the goal is to explain how time is allocated once on site, or during multipurpose or multisite trips, hours might be a better choice.

of units increases as if there were more data. For example, in a site-choice model, either an hour or a trip could be assumed to be a separate draw from the multinomial distribution. If all trips are of the same length (e.g., eight hours), the same parameter estimates will be obtained, but the estimated standard errors will be much smaller (and the *t*-statistics much larger) if the units are measured in hours. In our alternatives model, assuming 100 choice occasions seemed reasonable and leads to estimated *t*-statistics in the typical range. In the expenditures model, we assume each dollar is a draw from the multinomial distribution; dollars are the most common unit of account for expressing expenditures, and dollars are the units in which consumers think of many of the commodities purchased.³ We also report the *t*-statistics instead assuming \$10 units are multinomially distributed. Ten dollars might be the appropriate unit of account for commodities (fishing trips) that vary in cost from tens to hundreds of dollars. Assuming \$10 units are multinomially distributed causes the *t*-statistics to vary by roughly a factor of 4, and the estimated confidence intervals around the compensating variations are affected in a profound way. Both sets of *t*-statistics are unbiased, given the assumption made, and which one is more appropriate is an interesting issue for further research.

The General Form of the Alternatives Model

Assume that during the year there is a maximum of T_i occasions that individual i can fish. T_i reflects the time-intensive nature of fishing trips. During the year individual i chooses a vector of J alternatives, \mathbf{x}_i , where x_{ji} is the number of occasions individual i takes a trip to site j , $j = 1, \dots, J - 1$, and x_{ji} is the number of occasions the individual does something else $T_i = \sum_{j=1}^J x_{ji}$. Note that the magnitude of the time unit is the length of an occasion, and the length is simply defined such that no more than one trip can be taken per occasion.

Further assume the probability of observing x_i is

$$(1) \quad f(x_{1i}, x_{2i}, \dots, \mathbf{x}_{ji}; T_i, \theta_{1i}, \theta_{2i}, \dots, \theta_{ji})$$

$$= \frac{T_i!}{\prod_{j=1}^J x_{ji}!} \prod_{j=1}^J \theta_{ji}^{x_{ji}}$$

where $\theta_{1i}, \dots, \theta_{ji}$ are the probabilities of the distribution such that $0 < \theta_{ji} < 1 \forall j$, $\sum_{j=1}^J \theta_{ji} = 1$, and T_i is either fixed or an additional parameter. So θ_{ji} , $j = 1, \dots, J - 1$, is the proportion of occasions individual i hopes to fish at site j during the year. However, due to the vagaries of life and fishing trips, on each occasion the individual's observed activity is a random draw from a discrete distribution with θ_{ji} being individual i 's probability of drawing alternative j .

θ_i is assumed to be the vector of planned alternatives shares that maximizes individual i 's utility for the year. That is,

$$(2) \quad \theta_{ji} = \frac{x_{ji}^*}{\sum_{k=1}^J x_{ki}^*}, \quad j = 1, \dots, J$$

where $x_{ji}^* = x^*(B_i, p_i, \mathbf{a}_i)$ is individual i 's annual planned demand for alternative j . The vector of planned alternatives shares, θ_i , is obtained by maximizing $U(\mathbf{x}_i, \mathbf{a}_i)$ subject to $B_i = \sum_{j=1}^J p_{ji} x_{ji}$, where B_i is individual i 's yearly income, p_{ji} is individual i 's cost of a trip to site j , $j = 1, \dots, J - 1$, p_{ji} is his cost of doing something else, specifically, $p_{ji} = (B_i - \sum_{j=1}^{J-1} p_{ji} x_{ji}) / x_{ji}$, and $\mathbf{a}_i \equiv [a_{jki}]$, with a_{jki} the magnitude of attribute k individual i associates with alternative j , $k = 1, \dots, K$.

θ_i is determined ignoring the occasion constraint. The constraint is incorporated because the individual takes only T_i draws from the multinomial distribution. Note that even though θ_i remains constant across occasions, it is estimated under the assumption that the individual makes decisions with the whole year in mind. Each occasion is statistically independent, but θ_i is estimated assuming the individual plans all of the year's trips simultaneously at the beginning of the year to maximize utility for the entire year, taking account of across-occasion factors such as diminishing or increasing marginal utility and complementarities among trips. To contrast with a repeated RUM, the repeated RUM also assumes the probability of observing \mathbf{x}_i is described by equation (1), but assumes that the utility an individual receives from choosing alternative j on choice occasion c does not depend on what he plans to do on other choice occasions, so assumes away across-occasion factors such as diminishing or increasing marginal utility.

³ Expenditures on expensive commodities such as houses and cars are likely allocated in large units such as thousands of dollars.

The Data: Salmon Fishing in Maine and Canada

The data used are from a random sample of 168 Maine residents who held Maine Atlantic salmon fishing licenses (Morey, Rowe, and Watson). Data include complete trip records for a year and supporting data for Atlantic salmon trips to rivers throughout Maine and eastern Canada. To simplify the choice set, rivers are aggregated into eight distinct Atlantic salmon fishing areas: salmon rivers in Nova Scotia, salmon rivers in New Brunswick, salmon rivers in Quebec, and five separate river groups (watersheds) in Maine (the Machias, Dennys, Penobscot, Kennebec, and Saco groups).

Expected catch rates, reported in table 1, are significantly higher in Canada than at the Penobscot, and significantly higher at the Penobscot than at the four other Maine sites. Trip costs are also significantly higher for the distant Canadian sites. Average costs for sample trips are also reported in table 1. The alternatives model assumes 100 choice-occasions, so each individual gets equal weight in the likelihood function.⁴ Assuming each individual has the same number of choice occasions is common, but not necessary. The cost of the nonfishing alternative, p_{Ji} , varies significantly across individuals.⁵

The average number of salmon fishing trips is 17.05, with 11.87 of these trips to the Penobscot River. In total, the sample includes data on 2864 trips. Note the average angler takes a substantial number of trips, even though trips taken to Maine cost on average about \$200, and trips taken to Canada cost on average over \$800 (see table 1). Total trips and trips by river group are reported in table 2, as are observed aggregate trip shares for each of the eight fishing sites.

The NCES Preference Ordering and Resulting Alternatives Shares

For the salmon fishing example, the general form of the indirect utility function is

$$(3) \quad u_i = v(\mathbf{p}_i, B_i, \mathbf{catch})$$

where \mathbf{p}_i is the vector of nine prices (costs) for individual i and \mathbf{catch} is the vector of eight site-specific expected catch rates that do not vary across individuals or time. In particular, a NCES indirect utility function is adopted. NCES preferences are quite general but also relatively easy to estimate because the NCES expenditure function is *globally regular*, i.e., nondecreasing and concave in price space (Perroni and Rutherford 1995, 1996). Another desirable characteristic of the NCES is the potential for sites to be complements, which is not possible using a simple CES where all sites must be substitutes (Anderson and Moroney 1994). Perroni and Rutherford (1996) demonstrate that if all possible nesting structures are tested, the NCES is locally flexible. Comparing all possible nesting structures is feasible if the number of alternatives is small, but infeasible when there are nine alternatives, as is the case with this choice set. This potential for flexibility combined with global regularity gives the NCES an advantage over other flexible forms (e.g., the generalized Leontief, normalized quadratic, translog), which are typically not globally regular.

A nesting structure is adopted that assumes salmon fishing in Maine is separable from salmon fishing in Canada (a reasonable assumption because the catch rates and costs are grossly different between the two regions), and salmon fishing is separable from other activities. This grouping generates a three-level nest. This structure is consistent with the often-stated goals that aggregated alternatives should be more similar in terms of their characteristics (catch rates and travel costs) than are nonaggregated alternatives, and that elasticities of substitution within a group should vary less than elasticities across groups (Kazarian, Sato).

The nested CES allows elasticities of substitution to differ between groups of alternatives (Sato). Restricting the elasticities of substitution to be equal at every level of the nest reduces the nested CES to the simple CES (Verboven). The CES imposes a IIA-like property, whereas the NCES does

⁴ A total of 100 occasions is used rather than the sample maximum because if a much larger number were used, the behavior the model is meant to explain (fishing) would only be observed in a very small proportion of the total number of occasions. Most anglers take much fewer than 100 trips, although a very small number of anglers take many more than 100.

⁵ For the six individuals who took a hundred or more trips, p_{Ji} is undefined because $x_{Ji} = 0$, so p_{Ji} is approximated for these individuals by assuming $p_{Ji} = B_i - \sum_{j=1}^{J-1} p_{ji}x_{ji}$.

Table 1. Expected Catch Rates and Actual Fishing Costs for Eight Atlantic Salmon Fishing Sites

River Group	Expected Catch Rate per Trip ^a	Average Trip Costs ^b
<i>Maine Rivers</i>		
Penobscot	0.102	\$137
Machias	0.048	\$239
Dennys	0.058	\$246
Kennebec	0.074	\$203
Saco	0.039	\$288
<i>Canadian Rivers</i>		
Nova Scotia	0.948	\$806
New Brunswick	3.143	\$827
Quebec	2.360	\$885

^a Average of all angler's average trip catch rates.

^b Includes value of time; these averages are only for trips actually taken.

Table 2. Observed and Predicted Fishing Trips and Shares

	Observed Trips	Observed Shares	NCES Alternatives	NCES Expenditures	CES Alternatives	Repeated Nested Logit
All sites ^a	2864	1.000	2704	2613	2704	2749
Maine	2830	0.988	0.988	0.986	0.987	0.967
Canada	34	0.012	0.012	0.014	0.013	0.033
Penobscot	1994	0.696	0.654	0.724	0.667	0.645
Machias	544	0.190	0.077	0.042	0.083	0.071
Dennys	24	0.008	0.089	0.054	0.093	0.067
Kennebec	132	0.046	0.110	0.124	0.098	0.129
Saco	136	0.047	0.057	0.042	0.046	0.055
Nova Scotia	5	0.002	0.003	0.002	0.002	0.013
New Brunswick	17	0.006	0.005	0.005	0.006	0.011
Quebec	12	0.004	0.005	0.007	0.005	0.009

^a In this row, total predicted trips are reported for four models, rather than trip shares. All other rows report trip shares for the models. The nested-logit model underpredicts observed trips because of the choice occasion constraint (i.e., trips in excess of 100 were truncated). The number of observed trips after truncation is 2749.

not.⁶ Like the CES, the NCES maintains the assumption of homothetic preferences, so the alternatives shares are independent of the budget. Peronni (1992) has proposed a non-homothetic generalization of the NCES.

Specifically, assume the following NCES preference ordering:

$$(4) \quad u_i = v(P_{NFi}, P_{Fi}, B_i) = -e(P_{NFi}, P_{Fi})/B_i$$

where $e(P_{NFi}, P_{Fi}) = (P_{NFi}^{1-\sigma_P} + P_{Fi}^{1-\sigma_P})^{1/(1-\sigma_P)}$. P_{NF} and P_F are quality-adjusted price aggregators for fishing and nonfishing, respectively:

$P_{NFi} = p_{9i} \tilde{h}_{9i}$, $\tilde{h}_{9i} = h_{9i}^{\sigma_0/(1-\sigma_0)}$, and $P_{Fi} = (P_{Mi}^{1-\sigma_F} + P_{Ci}^{1-\sigma_F})^{1/(1-\sigma_F)}$. h_{9i} is the "quality" individual i associates with nonparticipation. It is a function of the characteristics of individual i . P_M and P_C are quality-adjusted price aggregators for Maine and Canada, respectively, in the second level of the nest: $P_{Mi} = (\sum_{j=1}^5 h_j^{\sigma_M} p_{ji}^{1-\sigma_M})^{1/1-\sigma_M}$ and $P_{Ci} = (\sum_{j=6}^8 h_j^{\sigma_C} p_{ji}^{1-\sigma_C})^{1/1-\sigma_C}$. h_j is the quality associated with site j , $j = 1, \dots, 8$, and is assumed to be a function of the site's expected catch rate.

σ_M and σ_C are the Hicks–McFadden (intraprocess) elasticities of substitution between Maine sites holding the number of trips to Maine constant, and between Canadian sites holding the number of trips to Canada constant. σ_F is the intraprocess

⁶ With the CES, the ratio of alternatives shares for two alternatives j and j' is a function only of the characteristics and prices of those two alternatives, whereas with the NCES if j is in the Maine group and j' is in the Canadian group, the ratio depends on the characteristics and prices of all of the fishing sites.

elasticity of substitution between Maine and Canadian fishing aggregates holding the number of fishing trips constant, and σ_p is the elasticity of substitution between fishing and nonparticipation (Anderson and Moroney 1993). σ_0 would be the intraprocess elasticity of substitution between different subcategories of nonparticipation, although these subcategories are not observed.

Given equation (4), the proportion of occasions individual i plans to fish is

$$(5) S_{Fi} = \frac{(1/P_{Fi})^{\sigma_p}}{(1/P_{Fi})^{\sigma_p} + (1/P_{NFi})^{\sigma_p}}$$

and the proportion for everything else is $S_{NFi} = 1 - S_{Fi}$. The proportion of fishing trips that individual i plans to choose a Maine site is

$$(6) S_{FMi} = \frac{(1/P_{Mi})^{\sigma_F}}{(1/P_{Mi})^{\sigma_F} + (1/P_{Ci})^{\sigma_F}}$$

and the proportion of fishing trips that individual i plans to choose a Canadian site is $S_{FCi} = 1 - S_{FMi}$. Thus, the proportion of choice occasions that individual i plans to fish in Maine is $S_{Mi} = S_{FMi} \times S_{Fi}$, and the proportion of choice occasions that individual i plans to fish in Canada is $S_{CCi} = S_{FCi} \times S_{CFi}$. The proportion of the fishing trips to Maine that individual i plans to site j is

$$(7) S_{Mji} = \frac{(h_i/p_{ji})^{\sigma_M}}{\sum_{k=1}^5 (h_k/p_{ki})^{\sigma_M}}$$

and the proportion of fishing trips to Canada that individual i plans to site j is

$$(8) S_{Cji} = \frac{(h_j/p_{ji})^{\sigma_C}}{\sum_{k=6}^8 (h_k/p_{ki})^{\sigma_C}}$$

Therefore, the proportion of choice occasions that individual i plans to site j in Maine is $S_{ji} = S_{Mji} \times S_{Mi}$, and the proportion of choice occasions that individual i plans to site j in Canada is $S_{ji} = S_{Cji} \times S_{Ci}$; so $\theta_{ji} = S_{ji}$.

To complete the specification, assume: $h_j = \exp(\alpha_0 + \alpha_{0C} + \alpha_1(\text{catch}_j) + \alpha_2(\text{catch}_j)^{-5})$, $j = 1, \dots, 8$, where α_{0C} is 1 if the site is a Canadian site ($j = 6, 7$, or 8) and zero otherwise; and $h_{9i} = \exp(\alpha_{0J} + \alpha_3 \text{age}_i + \alpha_4 \text{yrs}_i + \alpha_5 \text{club}_i + \alpha_6 (\text{yrs}_i)^{-5} + \alpha_7 (\text{age}_i)^{-5})$, where age is the angler's age, yrs is years of experience, and club is whether the angler belongs to a Penobscot fishing club.⁷

Another positive feature of this particular NCES model is that it allows the marginal rate of substitution (MRS) between fishing sites in different regions to vary across anglers, which is not a characteristic of the CES, because site characteristics are not individual-specific in the data. For CES preferences, the marginal utility for another trip to a site can be expressed as a function only of that site's characteristics, which are constant across anglers, and the number of trips taken to the site. Therefore, the marginal utility associated with the n th trip to the site is the same across anglers. In the NCES, however, marginal utility is a function of not only the quality of and number of trips taken to the site but also the trips taken to other sites, which vary across anglers.

An Estimated NCES Alternatives Model of Atlantic Salmon Fishing

The log of the likelihood function for the 168 Maine license holders,

$$(9) \ell = \sum_{i=1}^{168} \sum_{j=1}^9 x_{ji} \ln(\theta_{ji})$$

was maximized with respect to the elasticities of substitution and the α parameters in the h functions.⁸ σ_0 was severely multicollinear with σ_p , so σ_0 was set equal to σ_M . Further, allowing σ_C to differ from σ_M did not significantly improve the fit of the model, so σ_C was set equal to σ_M . The maximum likelihood estimates for the NCES are (with asymptotic t -statistics in parentheses): $\alpha_0 = 7.1110(16.4)$, $\alpha_{0C} = -3.9105(-13.4)$, $\alpha_1 = -1.5980(-8.2)$, $\alpha_2 = 4.7320(10.7)$, $\alpha_3 = -0.0391(-6.1)$, $\alpha_4 = 0.0653(5.0)$, $\alpha_5 = -0.1316(-6.8)$, $\alpha_6 = -0.4112(-6.9)$, $\alpha_7 = 0.4553(7.3)$, $\sigma_M = 2.0176(39.6)$, $\sigma_F = 0.8398(26.2)$, and $\sigma_p = 1.8819(32.3)$. As explained in the Introduction, the magnitude of these asymptotic t -statistics is a function of our assumption that each individual has 100 occasions.

The estimated elasticities of substitution indicate a desire for variety in one's choice

⁷ α_{0J} is fixed at 4.60517 for identification of the model, where $\exp(4.60517) = 100$.

⁸ As noted by Keller, and Pollak and Wales (1987), the NCES maximization algorithm can be numerically unstable: estimation requires estimating parameters that are exponents on other parameters that are also exponents. This potential for instability is minimized by scaling of the elasticities of substitution at the two highest levels so they change only 1% as fast as the other parameters. This sort of scaling is also often necessary when estimating the scale parameters in nested-logit models. Measures were taken to ensure that the estimated parameters result in a global, rather than a local, maximum.

of activities (fishing versus other activities), and in one's choice of fishing sites. Both cost and catch rates are important determinants of participation and site choice. In addition, age, years of experience, and whether the individual belongs to a Penobscot fishing club are important determinants of participation. The NCES elasticity estimates indicate that it is relatively easy to substitute between fishing sites in Maine or between sites in Canada, but more difficult to substitute trips across regions.

The CES ($\sigma_v = \sigma \forall v$) was also estimated, and the null hypothesis that preferences are CES is rejected. A repeated nested-logit model of participation and site choice (hereafter the RNL model) was also estimated. The RNL model was made as comparable as possible to the NCES alternatives model.⁹

Estimated total fishing trips for the sample and predicted trip shares (proportion of trips to site j) are reported in column 4 of table 2. The estimated total number of trips, 2704, is biased downward, probably because the model does not allow more than one hundred trips, a property it shares with the RNL model. The model accurately predicts the allocation of trips between Canada and Maine. The correlation between predicted total fishing trips for each individual and actual total fishing trips is 0.920, indicating that the alternatives model is explaining much of the variation in total trips across individuals.

The predicted NCES trip shares are similar to the observed shares, except for Machias, Dennys, and Kennebec, where it does not track well. The RNL model better predicts total fishing trips but does not do as well in predicting the allocation of those trips. For example, it overpredicts the share of fishing trips to Canada (0.012 actual and 0.033 predicted), which is likely because the RNL

model does not include a Canadian constant. The correlation between predicted total fishing trips for each individual and actual total fishing trips is 0.416, indicating that the RNL model is explaining much less of the variation in trips. This difference is likely because the NCES alternatives model allows for much more variation across anglers in terms of the MRS between salmon fishing and other activities. Given that the RNL model has scale parameters highly correlated with the eliminated Canadian constant, the exclusion of that variable is not the cause of the difference in the quality of the trip predictions. Even if the Canadian constant were included in the RNL model, and the model predicted the trip proportion to Canada better, the NCES alternatives model would still predict individual variation in trips much better.

The estimated NCES alternatives model is used to predict the impact of three different changes in resource availability at the Penobscot: elimination of Penobscot salmon fishing, doubling the Penobscot catch rate, and halving the Penobscot catch rate. All are possible scenarios. The predicted changes in trips by sample anglers when the Penobscot catch rate is doubled are reported in table 3, columns 3 and 4. The predicted increase in trips is slightly less than that predicted by the RNL model (columns 7 and 8). The NCES predicts an increase in trips to the Penobscot of 595, and an increase in total trips of 458, while the CES predicts increases of 1341 and 1225, respectively. The difference is largely because the CES grossly overestimates avid anglers' increase in trips in response to a doubling of the catch at the Penobscot, and slightly underestimates it for non-avid anglers, relative to the statistically superior NCES model.

The expected compensating variation, $E(CV)$, for individual i for a change from $\{p^0, catch^0\}$ to $\{p^1, catch^1\}$ is

$$E(CV_i) = B_i \left[1 - \frac{e(p_i^1, catch^1)}{e(p_i^0, catch^0)} \right].$$

Estimated $E(CV)$ s are reported in table 4 (columns 2-4) for the three scenarios. For example, mean $E(CV)$ for doubling the Penobscot catch rate is \$1,160 and the median is \$803. Remember that many salmon anglers spend significant amounts on salmon fishing, even though current catch rates are quite low. The mean $E(CV)$ s from the RNL model are, in absolute value, all significantly

⁹ It is a three-level model of participation and site choice with no income effects. It uses the same data set, incorporates the same linear and non-linear variables (except one), and has the same number of choice occasions as the NCES model. That is, the conditional indirect utility function for alternative j , $j = 1, \dots, 8$, is specified as $V_j = \exp(\alpha_0 + \alpha_{0c} + \alpha_1(catch_j) + \alpha_2(catch_j)^3) + \alpha_8(ppy_i - p_j)$, the same nonlinear form as used to denote quality, h_j , in the alternatives and expenditures models, except for the term dealing with units purchased of the numeraire (where ppy_i is per-period income for angler i and is defined as B_i divided by the number of periods). The same is true for the nonparticipation alternative. In terms of variables, the only difference is that a Canadian constant could not be estimated in the RNL model because this parameter and the scale parameters were highly correlated. See Morey, Rowe, and Watson for other examples of logit and NL models estimated using this data set.

Table 3. Predicted Mean and Median Changes in Behavior when the Penobscot Catch Rate is Doubled

Predicted trips	Observed	NCES Alternatives		NCES Expenditures		Repeated Nested Logit	
		Before	After	Before	After	Before	After
Total	17.05 (6.0)	16.10 (8.79)	18.82 (11.64)	15.55 (12.28)	16.73 (12.30)	16.36 (13.72)	19.60 (17.68)
Penobscot	11.87 (2.0)	10.53 (4.67)	14.07 (7.69)	11.26 (8.30)	14.74 (10.96)	10.55 (7.32)	14.63 (11.18)
Non-Penobscot	5.18 (1.0)	5.56 (2.12)	4.74 (1.16)	4.29 (1.52)	1.98 (0.63)	5.81 (3.47)	4.97 (2.27)

Note: The numbers in parentheses are the medians.

less than those from the NCES for all three scenarios, as are the ranges of the $E(CV)$ s across the sample. The difference in welfare estimates highlights the importance of incorporating the desire for variety into the model. The marginal utility from another trip is constant in the nested-logit model, whereas it is higher for initial trips and lower for subsequent trips in the NCES models. The mean of the $E(CV)$ s from the NCES and the CES are similar for a doubling of the Penobscot catch rate (\$1,160 versus \$1,064), but not as similar for the other scenarios. The medians are drastically lower from the NCES, as is the range of the $E(CV)$ s across the sample. This is because for avid anglers the CES model overestimates the utility increase resulting from a doubling of the Penobscot catch rate and underestimates it for nonavid anglers.

The Maine aggregate was found to be a gross complement to the Canadian aggregate for the sample (although Canada and Maine are net substitutes), an impossibility in the nested-logit model and CES.¹⁰ In the NCES, when the Canadian site qualities improve or their trip costs fall, anglers become more avid, substituting out of other activities and into fishing. Not only do trips to Canadian sites increase, but trips to the Maine region increase as well. How the estimated $E(CV)$ s for changes in the Penobscot catch rate are influenced by the quality of the

Canadian sites is drastically influenced by this complementarity.

For example, suppose the qualities at all three of the Canadian sites were twice their current levels. The mean CES $E(CV)$ for a doubling of the Penobscot catch rate would be 5% less than when the Canadian sites are at their current levels. Conversely, the mean NCES $E(CV)$ for a doubling of the Penobscot catch rate would be 65% more than when the Canadian sites are at their current levels. Because it does not admit complements, the CES provides the opposite results.

The NCES alternatives share model is quite pliant and could be extended in a number of directions. The Peronni (1992) generalization could be used to incorporate income effects. Other separability assumptions and nesting structures (including nonseparability) could also be investigated, or nonnested tests to compare alternative nonnested specifications could be used (Davidson and MacKinnon, Pollak and Wales 1991).

An Estimated NCES Expenditures Model for Atlantic Salmon Fishing

For the expenditures model, redefine x_j as the units purchased of a numeraire good, where all income not spent on fishing trips is spent on this numeraire. In this case, p_j is the price of the numeraire. Assume that everyone faces the same price for the numeraire; so, without loss of generality, set $p_j = \$1 \forall i$, which implies $p_{j_i} x_{j_i} = x_{j_i}$. In the expenditures model there is no fixed number of occasions, and each individual's contribution to the likelihood function is an increasing function of income. This feature is appropriate, because the intent is to explain the allocation

¹⁰ Note that a two-level model of region and site choice (i.e., a partial model omitting the participation decision) would not allow Maine and Canada to be complements, because aggregates at the highest level must be substitutes, and sites must be in the same subgroup to be complements (Anderson and Moroney 1993). In the three-level model Maine and Canada fishing aggregates can be complements because they are in the same middle fishing subgroup. A relatively large elasticity of substitution at the top participation level relative to the middle regional level drives the complementarity.

Table 4. Summary Statistics for the Estimated Expected Compensating Variations Associated with Three Different Changes in Species Availability at the Penobscot

Model	NCES Alternatives			NCES Expenditures ^a			Repeated Nested Logit		
	Mean	Median	Sample Range	Mean	Median	Sample Range	Mean	Median	Sample Range
Double catch	\$1160	\$803	\$98 to	\$1172	\$886	\$148 to	\$523	\$383	\$0.18 to
Rate	(217.4)		\$5,070	(58.11, 2389.69)		\$4811	(65.75)		\$2227
Half catch	-\$697	-\$490	-\$30 to	-\$597	-\$421	-\$34 to	-\$293	-\$200	-\$0.07 to
Rate	(135.4)		-\$3,066	(25.41, 1757.79)		-\$2858	(29.43)		-\$1402
Eliminate	-\$1768	-\$1,199	-\$51 to	-\$1,064	-\$688	-\$47 to	-\$891	-\$556	-\$0.15 to
Penobscot	(267.4)		-\$10,038	(52.62, 4026.21)		-\$5078	(30.95)		-\$4800

Note: The numbers in parentheses are the estimated standard errors of the mean estimated E(CV)s simulated using 300 random draws based on the estimated covariance matrix of the parameters.

^aThe first standard error for the expenditures model is based on the assumption that single dollars are multinomially distributed; the second standard error is based on the assumption that \$10 units are multinomially distributed.

of money, and different people have different amounts.¹¹

Assume the probability of observing a vector of dollar expenditures, $(p_{1i}x_{1i}, p_{2i}x_{2i}, \dots, p_{ji}x_{ji})$, is

$$(10) \quad f(p_{1i}x_{1i}, p_{2i}x_{2i}, \dots, p_{ji}x_{ji}; B_i, \theta_{1i}, \theta_{2i}, \dots, \theta_{ji}) = \frac{B_i!}{\prod_{j=1}^J (p_{ji}x_{ji})!} \prod_{j=1}^J \theta_{ji}^{p_{ji}x_{ji}}$$

As explained in the Introduction, $p_{ji}x_{ji}$ can be expressed in any monetary units. The choice does not affect the parameter estimates, but it does affect their asymptotic t -statistics. Initially, we expressed expenditures in single dollar units because it seemed most natural, but this causes the parameter estimates to have, for good or bad, large t -statistics. As a result, we also report the t -statistics with expenditures expressed in \$10 units. The choice of denomination depends on whether the researcher believes anglers allocate expenditures in single dollars, \$10 units, or some other denomination.

In the expenditures model, θ_{ji} , $j = 1, \dots, J - 1$, is the proportion of income that individual i hopes to spend on trips to site j . However, due the vagaries of life, actual expenditures deviate from planned expenditures such that there is only a θ_{ji} probability that each dollar will be spent on alternative j .

θ_i is assumed to be the vector of planned expenditures shares that maximizes individual i 's utility for the year. That is, for the expenditures model,

$$(11) \quad \theta_{ji} = \frac{p_{ji}x_{ji}^*}{\sum_{k=1}^J p_{ki}x_{ki}^*} = \frac{p_{ji}x_{ji}^*}{B_i}, \quad j = 1, \dots, J$$

where $x_{ji}^* = x^*(B_i, \mathbf{p}_i, \mathbf{a}_i)$ is individual i 's annual planned demand for alternative j . The planned expenditures shares, θ_i , are obtained by maximizing subject to $U(\mathbf{x}_i, \mathbf{a}_i)$ subject to $B_i = \sum_{j=1}^J p_{ji}x_{ji}$.

Assume the same NCES preference ordering as in the NCES alternatives model. Like the alternatives share for alternative j , $j = 1, \dots, 8$, the expenditures share can be expressed as the proportion of income the

individual plans to spend on fishing, multiplied by the proportion of the fishing budget the individual plans to spend fishing Maine (Canada), multiplied by the proportion of the Maine (Canada) fishing budget the individual plans to allocate to site j . For example, the proportion of the Maine fishing budget that individual i plans to spend on site j , $j = 1, \dots, 5$ is

$$(12) \quad S_{Mji} = \frac{p_{ji}(h_j/p_{ji})^{\sigma_M}}{\sum_{k=1}^5 p_{ki}(h_k/p_{ki})^{\sigma_M}}, \quad j = 1, \dots, 5.$$

The log of the likelihood function for the 168 Maine license holders is

$$(13) \quad \ell = \sum_{i=1}^{168} \sum_{j=1}^9 (p_{ji}x_{ji}) \ln(\theta_{ji}).$$

As with the NCES alternatives model and because of multicollinearity, the restriction $\sigma_0 = \sigma_C = \sigma_M$ is imposed in estimation. The maximum likelihood estimates for the NCES expenditures model are (with asymptotic t -statistics in parentheses—the first assuming dollars are multinomially distributed, and the second assuming \$10 units are multinomially distributed): $\alpha_0 = -12.454(-5.1, -1.2)$, $\alpha_{0C} = 65.1421(29.0, 7.4)$, $\alpha_1 = -2.6432(-195.3, -60.0)$, $\alpha_2 = 7.7560(214.1, 66.8)$, $\alpha_3 = -3.1275(-3.1, -0.8)$, $\alpha_4 = 0.0537(2.5, 0.742)$, $\alpha_5 = -3.1199(-8.7, -2.0)$, $\alpha_6 = -3.4837(-8.8, -2.0)$, $\alpha_7 = 0.8558(5.2, 1.3)$, $\sigma_M = 2.3700(528.4, 166.5)$, $\sigma_F = 0.9843(1914.6, 493.3)$, and $\sigma_P = 1.0961(99.3, 23.4)$. Comparing the expenditures model parameter estimates with the alternatives model parameter estimates indicates both significant similarities and significant differences. Differences are not surprising given that one model is explaining expenditures by site, and the other is explaining trips by site. In addition, the specification of x_{gi} differs, and in the alternatives model, the trip vector has a multinomial distribution, while in the expenditures model the expenditures vector has a multinomial distribution.

The NCES expenditures model predicts that the Maine and Canadian aggregates are gross complements to each other, whereas only Maine is a gross complement in the alternatives model. The estimated total trips and trip shares for the expenditures model

¹¹ Site-choice models also have this property of giving different weights to different individuals. In site-choice models, the contribution to the likelihood is a function of total trips.

are summarized in column 5 of table 2.¹² They differ from those of the alternatives model. Even though the expenditures model imposes no upper bound on the number of trips, the estimated total number of trips, 2613, is an underestimate, and even more so than the alternatives-model estimate. Like the alternatives model, the expenditures model accurately predicts the allocation of trips between Canada and Maine.

The correlation between predicted total fishing trips for each individual and actual total fishing trips for each individual is 0.372; recollect that it was 0.920 for the alternatives model. The expenditures model is not explaining the variation in individual trip-taking behavior as well as the alternatives model, but this is not surprising since it is designed to explain the allocation of expenditures by site rather than the allocation of trips by site. The correlation between the predicted and actual budgets is 0.389.

Table 3 (columns 5 and 6) summarizes how doubling the Penobscot catch affects participation and site choice. The estimated $E(CV_i)$ s for each scenario are summarized in table 4 (columns 5–7). The formula for calculating these $E(CV)$ s remains the same; the differences from the alternatives-model estimates result from the different assumption about p_{9i} and the different parameter estimates. The estimates for doubling and halving the Penobscot catch rate are similar to those from the alternatives model, but from the expenditures model the $E(CV)$ s for eliminating the Penobscot are significantly smaller in absolute value.

As with the alternatives model, the expenditures model predicts larger $E(CV)$ s in absolute value than does the RNL model. The NCES alternatives model and the NCES expenditures model both generate more variation in $E(CV)$ s across the sample than does the RNL model, and this depresses the RNL mean and median estimates relative to those from the NCES models.

Finally, note the standard errors on the $E(CV)$ s from the expenditures model, and how they differ as a function of whether dollars or \$10 units are assumed for the multinomial distribution. When one adopts the assumption that dollars are distributed in \$10 units, the mean of the estimated $E(CV)$ s

for each scenario is not significantly different from zero. The choice of units is important in theory and practice, and research needs to be done to determine the units in which individuals allocate money across alternatives. It is likely to be a function of the type and cost of the alternatives considered.

Conclusions

The alternatives model and expenditures model developed here are viable options for estimating recreational participation and site choice, both doing a good job of tracking observed behavior. Comparing a repeated discrete-choice model with these two models indicates that model choice matters, more in terms of the estimated compensating variations and demand responses associated with changes in quality than in terms of explaining observed aggregate behavior. The alternatives model does the best job of explaining variation in trips across individuals. This is not to suggest the repeated RUM should be abandoned, only that the alternatives model and the expenditures model are viable alternatives for explaining participation and site choice.

The alternatives model and the expenditures model are attractive in that they do not impose the restrictive assumption that where one goes on a given trip is independent of where one plans to go on other occasions. One can estimate the degree of preference for variety in the choice of sites and choose a utility function that allows complements, neither of which is possible with a repeated RUM. The empirical results indicate diminishing marginal utility associated with salmon fishing, a desire for variety in one's choice of sites, and that Maine and Canadian sites are complements.

Choosing between the alternatives model and the expenditures model depends, in part, on whether one is more comfortable with the assumption that budget not spent on salmon fishing is spent on a numeraire with a common price, or the assumption that money not spent on salmon fishing is spent on some other activity, the nature and cost of which varies across individuals. The expenditures model makes the former assumption, and the alternatives model makes the latter.

The distributional assumptions, that the trip vector is multinomially distributed in

¹² Examining the actual and estimated expenditures shares: 5.6% of total expenditures were on fishing trips, and the expenditures model predicts 5.3%; 87.14% of fishing expenditures were for Maine sites, and the expenditures model predicts 86.24%.

the alternatives model and that the expenditures vector is multinomially distributed in the expenditures model, are more consistent with the properties of demand and expenditures than is the common assumption that demands are joint normally distributed. However, one could modify either model by making some other distributional assumption.

One can specify any functional form for the indirect utility function, subject only to the limitations of empirical tractability. The estimated NCES preference ordering demonstrates some of the advantages of this ability to generalize, and highlights the generality of the NCES, including its ability to admit complements. In general, it is recommended that anyone who estimates demand systems considers the NCES; it has many desirable properties, and because it is globally regular, it is relatively easy to estimate.

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