APPENDIX D
MODEL VARIATIONS

This appendix presents model variations and reports estimation results. In Section D.1 models explaining choices from the alternative pairs using only the SP data are derived. These models assume both homogeneous and heterogeneous preferences. In Section D.2, heterogeneous preferences are introduced into the full model covered in Appendix B. A general finding in all of these variations is that using only SP choice data or allowing for preferences to vary across individuals has little effect on estimated mean damages from FCAs — in most models mean damages do not vary significantly from each other. The notable exceptions are the random parameters specifications and a specification that allows the marginal utility of money to vary with individual characteristics, both giving higher damage estimates.

Estimated mean compensating variations for many of the models discussed in detail below are summarized in Table D-1 for a change in FCA level from Level 4 to Level 1 (no FCAs). The estimated mean CV^G is reported for all models,[1] and the estimated mean E(CV^F) is reported for the models allowing substitution in and out of Green Bay to other sites. Recall that the estimated mean CV^G for the full model with no heterogeneity is \$9.75, and the estimated mean E(CV^F) is \$4.17. Along with mean consumer surplus, 95% confidence intervals for the means are reported. These ranges were simulated using 500 parameter draws from the estimated variance-covariance matrix.

Mean predicted Green Bay days under current and baseline conditions are also presented in Table D-1 for all of the models allowing substitution from other sites. They are computed by multiplying the model predicted probability of fishing Green Bay under current conditions by observed 1998 open-water fishing days at all sites in 1998, so total days are held constant.[2] All models closely predict the current mean: over the sample, the mean is 10.0, and predictions range from 10.0 to 10.9. The models are roughly consistent in predicting how mean days will increase with cleanup. Predictions in increased days range from 0.4 to 1.2 days; that is, the increases in percentage terms are from 4% to 15%.

1. For the random parameters models, estimated mean E(CV^G) is reported.

2. If instead of using the 1998-level catch rates (recall that the catch rate is the reciprocal of the average time to catch a fish), the 13-year averages from 1986 to 1998 were used, predicted days and consumer surplus per fishing day would increase by about 16%. Therefore, using the 1998 levels results in conservative estimates of days and damages.

Table D-1
Summary of Estimated Mean Compensating Variations from Green Bay Models: FCA Level 4 to No FCAs

	Mean CV per Green Bay Fishing Day[a]			Mean E(CV) Fishing Days[a,d]			Predicted Green Bay Days	
	All[b]	**Near[c]**	**Far[c]**	**All[b]**	**Near[c]**	**Far[c]**	**Current[d]**	**Baseline[d]**
Full model								
No heterogeneity	**$9.75** ($0.0) [$8.06-$11.73]	NA	NA	**$4.17** ($0.0) [$3.41-$5.00]	NA	NA	10.0	11.4
Classic heterogeneity in V_O only[e]	$10.46 ($0.0) [$8.79-$12.35]	NA	NA	$4.49 ($0.72) [$3.96-$5.75]	$4.63	$3.59	10.1	11.4
A-B-other								
Classic heterogeneity in V_O only[e]	$9.57 ($0.0) [$7.57-$11.75]	NA	NA	$4.35 ($0.58) [$3.48-$5.26]	$4.45	$3.68	10.9	11.8
Classic heterogeneity in A-B parameters only[f]	$9.31 ($1.47) [$7.73-$11.68]	$9.20	$10.68	$4.16 ($1.55) [$3.46-$5.27]	$4.29	$3.40	10.4	11.6
A-B								
No heterogeneity	$10.29 ($0.0) [$8.10-$13.22]	NA	NA	NA	NA	NA	NA	NA
Classic heterogeneity[f]	$10.15 ($1.72) [$7.99-$12.51]	$10.23	$9.17	NA	NA	NA	NA	NA
Random heterogeneity (normal)[g]	$12.90 ($0.0) [$8.70-$15.75]	NA	NA	NA	NA	NA	NA	NA
Random heterogeneity (lognormal)[h]	$17.67 ($0.0) [$12.63-$24.72]	NA	NA	NA	NA	NA	NA	NA

a. Weighted average (weighting is by actual days at all sites or Green Bay days) in all models with classic heterogeneity.
b. Standard deviation across sample in parentheses. 95% numerically-simulated confidence interval in brackets.
c. "Near" means shortest driving distance is within the eight targeted counties: 73 mi. "Far" is for households farther than 73 mi.
d. The average time to catch a fish is set at the 1998 level for all species.
e. Utility for *other* (V_O) is a linear function of distance and gender. No classic heterogeneity in A-B parameters.
f. FCA and catch parameters are linear functions of distance and gender.
g. FCA and catch parameters are normally distributed. Results reported for Hermite quadrature using 9 evaluation points. Expected *CV* is reported.
h. FCA and catch parameters are lognormally distributed. Results reported for simulation using 500 draws. Expected *CV* is reported.

D.1 A-B MODELS USING ONLY THE SP DATA FROM THE A-B CHOICE QUESTIONS

In this section models are presented that explain only the choices from the pairs of Green Bay alternatives with different site characteristics, conditional on fishing Green Bay. Only the SP data on site selection from the choice pairs (A versus B) are used; none of the SP or RP data on planned or actual numbers of Green Bay days are used. These models are called A-B models, and while their parameter estimates are consistent, the omission of the additional data on days reduces efficiency of the estimates. Further, because the A-B model does not allow substitution out of Green Bay fishing to other sites, only the *CV* per Green Bay fishing day, CV^G, can be estimated, as was explained in Appendix C.

D.1.1 A-B Model with Homogeneous Preferences

Initially, consider an A-B model with preferences that do not vary across individuals. Assume that the indirect utility function for the choice pairs is identical to that of the full model developed in Appendix B (see Equation 1 in Appendix B). The likelihood function is simply the portion of the likelihood in Appendix A that explains choice of alternative from the SP pairs:

$$L(k_{ij}, i=1,\ldots,m, j=1,\ldots J | x_{ij}^1, x_{ij}^2, b, s_e) = \prod_{i=1}^{m}\prod_{j=1}^{J} P_{ij}^{k_{ij}} \tag{1}$$

To estimate this model, s_e was fixed at $\sqrt{1/2}$. As a result, the expression $\sqrt{2}s_e$ disappears from the likelihood. Parameter estimates and the asymptotic *t*-statistics are presented in Table D-2. The estimated model predicts approximately 73% of the pairs correctly, and all of the parameters are statistically significant with the expected sign. Also, the parameter estimates are similar to the A-B estimates from the full model (Appendix B). The calculation of CV^G for all A-B models with no income effects is explained in Appendix C.

For a change in FCAs from current Level 4 to no FCAs results in an estimated mean CV^G of \$10.29, which does not vary across anglers. This value is only 6% higher than the value from the full model of \$9.75. However, the reduction in efficiency in the A-B estimate is reflected in the simulated 95% confidence interval of the mean CV^G: \$8.10 to \$13.22 for the A-B model, as compared to \$8.13 to \$11.81. The confidence interval for the A-B model is wider on the high end.

Table D-2
Parameter Estimates from Nonrandom A-B Models

Parameter\Model	Homogeneity	Heterogeneity
	Estimated Parameters (asymptotic *t*-statistics)	
Homogeneous parameters		
β_y	0.0459 (16.891)	0.0466 (15.381)
β_p	-0.5211 (-16.673)	-0.4707 (-4.904)
β_t	-0.0281 (-8.732)	-0.0285 (-3.472)
β_w	-0.0363 (-11.703)	-0.0192 (-2.447)
β_b	-0.0310 (-9.998)	-0.0310 (-3.787)
β_{FCA2}	-0.1770 (-3.672)	-0.2351 (-3.517)
β_{FCA3}	-0.2437 (-4.942)	-0.3210 (-4.935)
β_{FCA4}	-0.4724 (-9.652)	-0.6388 (-9.219)
β_{FCA5}	-0.6698 (-13.703)	-0.8897 (-12.240)
β_{FCA6}	-0.4533 (-8.958)	-0.6030 (-8.975)
β_{FCA7}	-0.7890 (-17.484)	-1.0494 (-12.729)
β_{FCA8}	-1.0772 (-22.733)	-1.4622 (-13.316)
β_{FCA9}	-1.1872 (-21.733)	-1.5970 (-13.866)
Heterogeneous parameters		
β_{pg}		-0.1398 (-1.321)
β_{tg}		0.0031 (0.351)
β_{wg}		-0.0214 (-2.491)
β_{bg}		-0.0008 (-0.086)
β_{FCAg}		-0.2868 (-4.706)
β_{pd}		8.667e-4 (3.574)
β_{td}		-3.455e-5 (-1.594)
β_{wd}		2.424e-6 (0.109)
β_{bd}		1.091e-5 (0.598)
β_{FCAd}		-2.981e-4 (1.944)

D.1.2 Investigating Learning/Fatigue and Positioning Bias

The A-B model with no heterogeneity was then modified to examine whether *learning* or *fatigue* effects exist, and whether there is *positioning bias* toward the A (i.e., the first) alternative. As the respondent gains knowledge and understanding of the survey process, the learning effect may express itself through a decrease in the stochastic variance s_e^2 for initial choice pairs as compared to later ones. Recollect there was a practice pair that was not included in estimation. Similarly, if there are a large number of survey choice pairs, a fatigue effect may set in as the respondent tires of the data elicitation process. This effect may be manifested as an increase in noise for choice pairs toward the end of the process.

Two A-B methods were employed to investigate the presence of learning and fatigue. First, a model was estimated in which s_e was fixed at $\sqrt{1/2}$ for the middle four choice pairs ($j \in$ [3, 4, 5, 6]), but s_e was separately estimated as an unrestricted parameter for the first two choice pairs ($j \in$ [1, 2]) and the last two ($j \in$ [7, 8]). Results suggest weak but statistically insignificant learning and fatigue effects. As compared to 0.707 (= $\sqrt{1/2}$), estimated $s_{ej}, j \in [1,2],$ equals 0.936, and estimated $s_{ej}, j \in [7,8],$ equals 1.052. Based on a likelihood ratio test, the null hypothesis that variances are equal across the choice occasions cannot be rejected. CV^G for this model is $9.99, which is only 2% higher than the estimate from the model with all variances restricted to be the same.

The second method fixes s_e at $\sqrt{1/2}$ for all of the choice occasions, but allows b to vary in an unrestricted fashion over the three choice-occasion groups. This method is less restrictive than the previous method. If *parameter proportionality* holds, where b only varies across choice occasions by a factor of proportionality, learning and fatigue would be evident if the elements of b were all bigger for $j \in$ [3, 4, 5, 6] than for the first two or last two choices. Because s_e is fixed in estimation at the same value for all three groups, and because only the ratio of b and s_e is identified in estimation, more noise would show up as smaller values for b . The results from the three independently estimated models show that parameter proportionality does not hold, because b does not vary systematically across the choice occasions. A likelihood ratio test indicates that b is not proportional across choice occasions at conventional significance levels. However, there is some moderate evidence of fatigue, although no evidence of learning. The estimated CV^G, a weighted average across the choice-occasion groups, is $10.94, which is 12% higher than the estimate from the model restricting b to be the same across choice occasions. As a result, we maintain that $9.75 is a conservative estimate of damages per day. The CV^G estimates separately for the groups are $8.65 for the early choice occasions, $15.02 for the middle choice occasions, and $5.06 for the later choice occasions. Note that relative to the $9.75 estimate from the main model, these are all imprecise estimates.

Finally, an A-B model with no heterogeneity was estimated that included a dummy variable for whether the alternative selected was the first presented, alternative A. This variable was a statistically insignificant determinant of choice, so the null hypothesis of no positioning bias could not be rejected.

D.1.3 A-B Models with Heterogeneous Preferences: Interaction

In this section, A-B model parameters are allowed to interact with observed individual socioeconomic characteristics, the "classic" method of admitting heterogeneous preferences. Consequently, CV_i^G also varies as a function of characteristics. While marginal utilities for changes in site characteristics and consumer surplus vary in plausible ways as a function of individual attributes, we also find that estimated mean CV^G for the sample is quite comparable to that from the full model or to the A-B model with no heterogeneity. Incorporating heterogeneity at the A-B level appears to have little effect on mean damages (see Table D-1).

Preliminary analyses and simple statistics for the sample suggested that the A-B choices vary with gender and distance from Green Bay. Other socioeconomic characteristics were not as important in the preliminary analysis. Therefore, the effects of catch and FCAs were modeled as functions of those two variables. The set of FCA marginal utilities is assumed to vary proportionately as a function of individual characteristics. For example, FCA effects for men are all decreased by the percentage β_{FCAg} (see Equation 2 below) compared to women. The likelihood function is the same as in Section D.1.1, with the only exception that $V_{ij}^{k_{ij}}$ now includes interactions with individual characteristics. Specifically:

$$\begin{aligned} V_{ij}^{k_{ij}} &= b_y(-FEE_{jk}) + \sum_{l=1}^{4} ACT_{ljk}[b_l + b_{lg}(GEND_i) + b_{ld}(DIST_i)] \\ &+ \sum_{q=2}^{9} FCAq_{jk}[b_{FCAq} + b_{FCAg}b_{FCAq}(GEND_i) + b_{FCAd}b_{FCAq}(DIST_i)], \end{aligned} \tag{2}$$

where l indexes the fish species for catch (l = 1, ..., 4), ACT_l is the average time to catch species l, q indexes the FCA levels (q = 2, ..., 9; β_{FCA1} is fixed at zero for identification), FEE_{jk} is the launch fee for alternative k in pair j, $GEND_i$ equals one if angler i is a male, and $DIST_i$ is the closest distance to Green Bay from either angler i's vacation cabin or home.

This model was found to be statistically superior to the homogeneous A-B model on the basis of a likelihood ratio test, although the proportion of choices predicted correctly did not change appreciably. Parameter estimates are reported in Table D-2. The effects of FCAs and catch for some species were found to vary significantly as a function of gender and distance from Green Bay. Women were found to have larger FCA effects (in absolute value) and therefore larger damages.[3] They care more about FCAs than men. Conversely, men were found to have a larger

3. For example, the mean CV^G for a change from FCA Level 4 to no FCAs for individuals living five miles away is \$13.68 for women and \$9.75 for men. Note that the mean compensating variations for all classic heterogeneous models reported in this appendix are weighted means, where the weights used were either the individual's proportion of the sample number of days to Green Bay in 1998 for CV^G or the proportion of the sample number of days to all sites in 1998 for E(CV^F). These weights were used because we estimate CV per Green Bay fishing day or per fishing day, not per angler.

marginal utility for catching walleye. Marginal utilities for perch catch and FCAs decrease with distance, while the marginal utility for trout and salmon catch increases with distance at a marginal significance level (the *t*-statistic is -1.59). For those traveling to Green Bay from within the eight targeted counties (within 73 miles), the mean CV^G is \$10.23, while for those farther away it is \$9.17. Over the entire sample, the mean is \$10.15, with a standard deviation of \$1.72. The simulated 95% confidence interval for mean CV^G is \$7.99 to \$12.51. Note that this mean is only 4% higher than the mean from the full model.

Other interaction specifications were run as well, and while some of these generalizations were statistically significant, uniformly they do not lead to statistically or substantively different estimates of mean consumer surplus. For example, the effects of FCAs and catch on utility were also allowed to vary as a function of the angler's target species. The effects on utility of catch changes for all four of the target species are all greater for the respective target anglers, and perch and walleye anglers care more about FCAs than other target anglers or anglers who have no target.[4] The effect on damages was small, however. Effects on utility from FCAs and catch were not found to vary significantly as a function of the actual number of Green Bay days.[5] Finally, marginal utility of money was allowed to vary across gender and income groups; males and the wealthy have a significantly lower marginal utility of money. This specification led to a higher estimate of the weighted mean CV^G, \$12.36 (27% higher than \$9.75). However, the simulated confidence interval on mean CV^G was quite large, [-\$19.34 to \$35.98], because some draws of the price parameter for affluent males using the estimated covariance matrix are very small and even have the wrong sign. Therefore, it is not possible to conclude that \$12.36 is significantly higher than \$9.75.

D.1.4 A-B Model with Heterogeneous Preferences: Random Parameters

Two primary issues have motivated the use of random parameters in modeling consumer choice. First, random parameters provide a way to induce correlation in the stochastic components of utility within pairs of alternatives and across an individual's choice occasions. Hausman and Wise (1978) were the first to model explicitly correlated disturbances. Second, random parameters allow for preference heterogeneity across individuals without having to model heterogeneity explicitly as a function of individual covariates. Note that the estimates from our main model,

---

4. Note that target is correlated with distance (a much higher percentage of yellow perch anglers live close to Green Bay, and a much higher percentage of salmonid anglers live farther away). An angler is defined here as a target angler for a species if he fishes for that species "often" or "almost always," and does not fish for any other species "often" or "almost always."

5. Along these lines, omitting extremely avid anglers with a large number of days from the data set prior to estimation was also not found to have a notable or significant effect on parameter or per-day consumer surplus estimates.

which does not explicitly model correlation across pairs, are consistent in the presence of such correlation.

Random parameters have been used to model choice-experiment data for a wide array of commodities and environmental amenities, including alternative-fuel vehicles (Brownstone and Train, 1999); appliance efficiency (Revelt and Train, 1998); forest loss along the Colorado Front Range resulting from global climate change (Layton and Brown, 1998); and the level of preservation of marble monuments in Washington, DC (Morey and Rossmann, 1999). Three recreational site-choice examples using simulation with revealed preference data are a partial demand system of fishing site choice in Montana (Train, 1998), and complete demand systems of participation and site choice for Atlantic salmon fishing (Breffle and Morey, 1999) and fishing in the Wisconsin Great Lakes region (Phaneuf et al., 1998).

Model Specification

The random parameters A-B model for Green Bay fishing explicitly estimates the correlation between disturbances within pairs and across choice occasions, in the spirit of Hausman and Wise (1978). Assumption 2 from Appendix A is maintained, but assumption 1 is now replaced by:

$$b_i = b + u_i, u_i \text{ i.i.d.} \sim N(0, \Sigma) \tag{3}$$

where υ_i is an $L \times 1$ random vector that represents heterogeneity of preferences across individuals.[6] An individual's marginal utility of an alternative's characteristic differs from the average by an additive, mean-zero random variable assumed uncorrelated with the model disturbance. All J pairs are evaluated by the individual with these marginal utilities. Then:

$$U_{ij}^{k_{ij}} = b_i' x_{ij}^{k_{ij}} + e_{ij}^{k_{ij}} = b' x_{ij}^{k_{ij}} + (u_i' x_{ij}^{k_{ij}} + e_{ij}^{k_{ij}}), \tag{4}$$

where the new model disturbance is in parentheses. It is straightforward to find the correlation between these disturbances (and hence the utilities) within a pair and across pairings for each individual. Within a pair we have:

$$\mathrm{E}[(u_i' x_{ij}^1 + e_{ij}^1)(u_i' x_{ij}^2 + e_{ij}^2)] = (x_{ij}^1)' S\, (x_{ij}^2), \tag{5}$$

and from pair j to pair l we have:

$$\mathrm{E}[(u_i' x_{ij}^{k_{ij}} + e_{ij}^{k_{ij}})(u_i' x_{il}^{k_{il}} + e_{il}^{k_{il}})] = (x_{ij}^{k_{ij}})' S\, (x_{il}^{k_{il}}). \tag{6}$$

6. This is the usual formulation for the random coefficients model. See Hildreth and Houck (1968), Swamy (1970), and Hsiao (1975).

With correlation allowed, it is now more convenient for the unit of observation to be the individual (*i*), not the individual-pair (*i*, *j*) as in the nonrandom model. The probability of observing the vector of *J* pair-wise choices is a *J*-dimensional multinormal probability:

$$P_i = P(K_{i1} = k_{i1},\ldots,K_{iJ} = k_{ij}) = P(U_{i1}^{k_{i1}} > U_{i1}^{3-k_{i1}},\ldots,U_{iJ}^{k_{iJ}} > U_{iJ}^{3-k_{iJ}}). \tag{7}$$

Substituting the random utility model and the specification for the β_i (Equation 3) into Equation 7 yields, after some rearranging:

$$\begin{gathered} P_i = P(K_{i1} = k_{i1},\ldots,K_{iJ} = k_{ij}) = \\ P[(u_i' x_{i1}^{3-k_{i1}} + e_{i1}^{3-k_{i1}}) - (u_i' x_{i1}^{k_{i1}} + e_{i1}^{k_{i1}}) < -b'(x_{i1}^{3-k_{i1}} - x_{i1}^{k_{i1}}), \\ (u_i' x_{i2}^{3-k_{i1}} + e_{i2}^{3-k_{i1}}) - (u_i' x_{i2}^{k_{i1}} + e_{i2}^{k_{i1}}) < -b'(x_{i2}^{3-k_{i1}} - x_{i2}^{k_{i1}}), \\ \vdots \\ (u_i' x_{iJ}^{3-k_{iJ}} + e_{iJ}^{3-k_{iJ}}) - (u_i' x_{iJ}^{k_{iJ}} + e_{iJ}^{k_{iJ}}) < -b'(x_{iJ}^{3-k_{iJ}} - x_{iJ}^{k_{iJ}})]. \end{gathered} \tag{8}$$

Although evaluation of this integral is more complicated than the equivalent expression in the nonrandom model, the "equicorrelated" nature of the problem means that P_i can be calculated as the integral of a joint conditional probability over the density of υ_i by standard reasoning.[7] The *J* events are correlated, but the source of the correlation is the individual-specific parameter error vector υ_i. This common factor design allows for the computational simplification mentioned above. The *J* events in the probability in Equation 8, conditional on υ_i, are independent, so the joint conditional probability may be written as the product of the *J* conditional probabilities. Then the resulting product is integrated with respect to υ_i to remove the conditioning:

$$\begin{aligned} P_i = \int_{-\infty}^{\infty} & P[e_{i1}^{3-k_{i1}} - e_{i1}^{k_{i1}} < -b'(x_{i1}^{3-k_{i1}} - x_{i1}^{k_{i1}}) - u_i'(x_{i1}^{3-k_{i1}} - x_{i1}^{k_{i1}}), \\ & \vdots \\ & e_{iJ}^{3-k_{iJ}} - e_{iJ}^{k_{iJ}} < -b'(x_{iJ}^{3-k_{iJ}} - x_{iJ}^{k_{iJ}}) - u_i'(x_{iJ}^{3-k_{iJ}} - x_{iJ}^{k_{iJ}}) | u_i] f(u_i) du_i \\ = & \int_{-\infty}^{\infty} \prod_{j=1}^{J} P[e_{ij}^{3-k_{ij}} - e_{ij}^{k_{ij}} < -b'(x_{ij}^{3-k_{ij}} - x_{ij}^{k_{ij}}) - u_i'(x_{ij}^{3-k_{ij}} - x_{ij}^{k_{ij}}) | u_i] f(u_i) du_i \\ = & \int_{-\infty}^{\infty} \prod_{j=1}^{J} \Phi[(-\frac{b + u_i}{\sqrt{2} s_e})'(x_{ij}^{3-k_{ij}} - x_{ij}^{k_{ij}})] f(u_i) du_i \end{aligned} \tag{9}$$

where ϕ is the *L*-variate multinormal density function with mean vector **0** and covariance matrix $\sum$:

7. See Butler and Moffitt (1982) and Waldman (1985).

$$f(u_i) = \{2p\,|\Sigma|\}^{-L/2} \exp\left[-\tfrac{1}{2} u_i' \Sigma^{-1} u_i\right]. \tag{10}$$

The order of magnitude of the integral in Equation 9 is determined by the assumptions made for Var(υ_i) = $\sum$. Specifically, it is equal to the number of distinct nonzero diagonal elements, which is the number of parameters assumed to be random.[8]

For the model of Equation 3, $\beta_i = \beta + \upsilon_i$, the likelihood of observing $k_{i1}, \ldots, k_{iJ}$ is:

$$L(k_{ij,} i = 1,\ldots,m, j = 1,\ldots, J \,|\, x_{ij}^1, x_{ij}^2; b, s_e, \Sigma) = \prod_{i=1}^{m} P_i \tag{11}$$

where the P_i are from Equation 9.

Methods of Estimation

For the purpose of estimation by maximum likelihood, Equation 11 can be evaluated in either of two ways. First, since the kernal of $\phi(\cdot)$ is of the form exp(-[$\cdot$]^2), the combination of Equations 9 and 10 with a change of variables ($v = u / \sqrt{2}s_u$ for the case of one random parameter) can be written in the form:

$$\int_{-\infty}^{\infty} e^{-n^2} g(n)\,dn. \tag{12}$$

This integral can be approximated using Hermite polynomial quadrature, which is fast enough to be a practical computational method (Butler and Moffit, 1982; Waldman, 1985). Quadrature can be made as accurate as necessary. If the order of magnitude of the integral is small, which is the case in the current application [in Hausman and Wise (1978), three parameters are random], the estimation problem is computationally tractable by quadrature. Second, if quadrature is not feasible because the order of magnitude is too large, a simulation method could be used (see Layton and Brown, 1998; Train, 1998; Breffle and Morey, 1999). Using simulation, the integral is approximated in two steps: first, the joint probability is computed many times using a large number of random draws from the distribution of υ, and then the average is computed. Details on Hermite quadrature are relegated to Section D.3.

8. Under normality and the additional assumption of a diagonal $\sum$, the multinormal joint density of υ_i, $\phi(\upsilon_i)$, factors into the product over k of $\phi(\upsilon_{ik})$, although no further simplification appears to be possible because each element of υ_i appears in each probability. This means that there is no computational advantage in the additional assumption of a diagonal covariance matrix.

Estimated Random Parameters A-B Model

The four parameters on the catch rates and the eight parameters on the FCA dummy variables are all random, and assumed to be normally distributed with zero covariance. There is no classic heterogeneity in the model. In addition, it is assumed the standard deviations of the catch parameters vary in proportion to their means, and the same is separately true for the FCA parameters. That is, the ratio of the mean parameter to the standard deviation is the same for each of the four catch rates, and for each of the eight FCA levels. Therefore, only two standard deviations are estimated. Assuming that the standard deviation varies in proportion to the mean is a common way of dealing with heteroskedasticity, and allows the model to be more general without making it intractable. This technique is similar to one used by Brownstone and Train (1999) in a random parameters logit model, where the standard deviation was assumed to be equal to the mean. The marginal utility of money is not assumed to be random due to undesirable effects on the distribution of the E(*CV*)s because the price parameter is in the denominator of the *CV* formula (Layton and Brown, 1998; Phaneuf et al., 1998).

Specifically, the conditional indirect utility function for alternative *j* in angler *i*'s *k*-th choice is:

$$V_{ij}^{k_{ij}} = b_y(-FEE_{jk}) + (b_c + n_{ci})\sum_{l=1}^{4} b_l(ACT_{ljk}) + (b_{FCA} + n_{FCAi})\sum_{q=2}^{9} b_{FCAq}(FCAq_{jk}) \qquad (13)$$

where β_c and β_{FCA} are the mean base catch and FCA parameters, respectively; β_l is the deterministic multiplicative factor shifting the mean (and the standard deviation of the random component) of each catch parameters for the four species; β_{FCAq} is the multiplicative factor for FCA level *q*; and β_y is the marginal utility of money. The base standard deviations of the catch and FCA parameters are σ_c and σ_{FCA}, and β_p (for perch) and β_{FCA2} are fixed at one to achieve identification of the model.

This model was estimated using both quadrature and simulation, and parameter estimates are reported in Table D-3. Likelihood ratio tests show that the randomization of the catch and FCA parameters significantly improves model fit relative to the homogenous nonrandom A-B model. Results from various model runs show that 500 draws in simulation and 9 evaluation points (see Section D.3) using quadrature are sufficient for parameter estimates to be stable. That is, at these levels of draws and points, parameter estimates are the same within 2%, and parameters do not change significantly with more draws or evaluation points. Therefore, there is virtually no simulation noise. An interesting finding is that simulation took over three times as much computer time as quadrature for the 2% threshold.

The ratios of the standard deviation to the mean are 0.66 and 0.92, which match well with the ratios for random parameters in other studies valuing environmental improvements. The range over 20 parameters in 3 studies is 0.40 to 14.29, with a mean of 2.28 and a median of 1.43 (Layton and Brown, 1998; Phaneuf et al., 1998; and Train, 1998). The estimated parameters of

Table D-3
Parameters[a] and Consumer Surplus Estimates for Random Parameters A-B Model

Method	Hermite Quadrature	Simulation
Evaluation points/ random draws	9	500
Mean parameters		
β_y	0.0555 (15.267)	0.0556 (15.282)
β_c	-0.645 (-11.607)	-0.648 (-11.607)
β_{FCA}	-0.327 (-4.916)	-0.324 (-4.513)
β_p	1.0 (fixed)	1.0 (fixed)
β_t	0.0480 (6.384)	0.0478 (6.348)
β_w	0.0647 (7.985)	0.0650 (7.989)
β_b	0.0544 (7.295)	0.0544 (7.306)
β_{FCA2}	1.0 (fixed)	1.0 (fixed)
β_{FCA3}	1.618 (6.224)	1.643 (5.774)
β_{FCA4}	2.189 (6.519)	2.215 (5.938)
β_{FCA5}	2.963 (6.151)	3.000 (5.608)
β_{FCA6}	2.463 (5.944)	2.503 (5.437)
β_{FCA7}	3.531 (5.857)	3.578 (5.326)
β_{FCA8}	4.813 (5.607)	4.881 (5.098)
β_{FCA9}	5.300 (5.526)	5.384 (5.035)
Standard deviations[b]		
σ_c	0.428 (-5.270)	0.431 (-5.322)
σ_{FCA}	0.302 (-5.638)	0.296 (-5.238)
E(*CV*)s No FCAs v. *FCA*4	$12.90	$12.89

a. Asymptotic *t*-statistics are reported in parentheses.
b. *t*-statistics are for the natural logarithms of the standard deviations. The parameters were exponentiated in estimation to restrict them to be positive.

the normal distributions also imply that 6.6% of the population have catch parameters of the opposite sign (i.e., they value catch reductions) and 14.0% have FCA parameters of the opposite sign.[9] This result is an artifact of the distributional assumption.

E(CV^G) was estimated for a change to no FCAs from FCA Level 4.[10] The computation of E(CV^G) for a random parameters model with no income effects and only one alternative in each state of the world, such as this model, is straightforward. It can be computed as the difference between utility in the two states divided by the marginal utility of money. Because utility is linear in β, the formula for E(CV_i^G) when some parameters are random (but not the price parameter) and there is only one alternative in each state is the same as for the nonrandom model:[11]

$$\begin{aligned} \mathrm{E}(CV_i^G) &= \int_{-\infty}^{\infty} \frac{1}{b_y}\left[b_i'(x_i^1 - x_i^0)\right] f(n_i)\, dn_i \\ &= \frac{1}{b_y}\left[b'(x_i^1 - x_i^0)\right], \end{aligned} \tag{14}$$

where β is the vector of the means of the parameters.[12]

Note that because the choice of alternatives is not modeled as a function of individual characteristics, E(CV_i^G) = E(CV) ∀ i. The estimated E(CV^G)s for the two approximation methods are also reported in Table D-3 with the parameter estimates. Estimated E(CV^G) is $12.90 using the model estimated by quadrature, which is higher than $9.75 from the nonrandom model with RP data. The mean parameters for FCAs are about 20% larger than the estimates from the nonrandom model, generating higher damages. It is reasonable to expect that making parameters random may significantly raise or lower E(CV).

The normal specification of υ is only one possibility from many choices. A second random parameters A-B model was estimated under the assumption that the random parameters are lognormally distributed: $\ln(b_{ci}) \sim \mathrm{N}(b_c, s_c)$ and $\ln(b_{FCAi}) \sim \mathrm{N}(b_{FCA}, s_{FCA})$. This distributional assumption restricts the marginal utilities for increases in the time it takes to catch fish and the severity of FCAs to be negative to everybody. Because Hermite quadrature only applies when the distribution is normal, the simulation method was used with 500 draws. The estimated distributions of $\ln(b_C)$ and $\ln(b_{FCA})$ are -1 × N(-0.598, 0.670) and -1 × N(-1.560, 1.286),

9. Because the standard deviations of all random catch parameters are restricted to vary proportionately with their means, and the same is true for FCA parameters, these percentages apply to all catch and FCA parameters, respectively.

10. Note that with random parameters, CV_i^G is a random variable which depends on u_i.

11. In a multi-site random model, E(CV) would need to be numerically approximated just as the joint probability.

12. If a parameter is not random, its value equals the mean for all individuals.

respectively. The estimated E(CV_G) is \$17.67, which is considerably larger than \$9.67. The larger value is not surprising since the mean of a lognormally-distributed random parameter b_i is an increasing function not only of the mean b but also the standard deviation s : $\mathrm{E}(b_i) = \exp(b + (s^2/2))$. The mean β_{FCAi} is -0.480 when the distribution is assumed to be lognormal, as compared to -0.327 under the normal assumption. We do not estimate a model in which the price parameter is random.[13]

D.2 VARIATIONS ON MODELS ALLOWING SUBSTITUTION TO OTHER SITES

Classic heterogeneity is incorporated into models allowing substitution in and out of Green Bay in two ways. In the first of these models, the same specification for $V_{ij}^{k_{ij}}$ was used as presented in Equation 2, where the marginal utilities from FCAs and catch are assumed to be functions of gender and distance. This model was estimated using the SP data from the choice pairs, the expected number of days the chosen alternative would be visited from the followup questions to the pairs, and the RP data on total number of days.[14] This model is referred to as an *A-B-other* model in Table D-1.[15] In the second of the two models, the A-B parameters are assumed to be homogeneous, but the utility for *other*, V_{Oi}, is assumed to be a function of gender and distance:

$$V_{Oi} = b_0 + g_1 GEND_i + g_2 DIST_i. \tag{15}$$

---

13. Note that mean E(CV^G) would have to be simulated if the price parameter is random because the formula is nonlinear in the price parameter. Train (1998) allows the price parameter to be random and lognormally distributed. Layton and Brown (1998), however, warn of undesirable effects on the distribution of the E(*CV*)s as a result (because the price parameter is in the denominator of the *CV* formula), and hold the price parameter fixed. Phaneuf et al. (1998) also hold the marginal utility of money fixed. A small draw of the price parameter from its distribution will cause the E(*CV*) associated with that draw to balloon, which overall will have an upward effect on simulated mean E(*CV*).

14. When the RP data on 1998 Green Bay days were included, the model did not converge. This is not surprising, and nonconvergence does not detract from the quality of estimates from the main model with homogeneous preferences or any of the convergent models with heterogeneity. Introducing a large number of additional variables into a model often results in multicollinearity. As a result, parameters cannot be estimated with precision. The covariance matrix computed as the inverse of the Hessian matrix of numerical approximations of second order partial derivatives of the log-likelihood will not in fact invert if the Hessian is nearly singular (i.e., the likelihood function is virtually flat in some dimensions).

15. Results for another A-B-other model, which allows classic heterogeneity in V_o, are also reported in Table B-1. The parameter and consumer surplus estimates from A-B-other models are similar to the main model. These estimates are consistent but less efficient than the main model because they do not include the RP data on Green Bay days.

In this second specification, the RP data on actual Green Bay days is included. Both of these generalizations significantly increase explanatory power. We were unable to get convergence on a model using all the data with classic heterogeneity in both $V_{ij}^{k_{ij}}$ and V_{Oi}.

Parameters from the A-B-other model with classic heterogeneity are reported in Table D-4 and show similarities and differences when compared to the A-B model with classic heterogeneity. Again, we find that women have a higher WTP for eliminating FCAs in Green Bay. Men have significantly higher values for increasing catch rates for all species than do women.[16] An important difference in the results is that both FCA and catch effects are larger (in absolute value) for anglers at a greater distance. Also, the parameters from the A-B-other model with heterogeneity have much higher *t*-statistics than the parameters from the A-B model with heterogeneity. However, the estimated mean CV^G of $9.31 is similar to the full model without heterogeneity; it is less than 5% lower.

Also, the amount of noise in the stochastic term for the other index can be compared to that from the Green Bay choice pairs, because they are assumed to be uncorrelated, and s_0^2 is separately estimated. A greater level of randomness is expected for the other site because explicit characteristics of the site are not included in the model. The estimate of s_0^2 is over 10, which is greater than ½, the value of s_e^2.

Results from the full model with heterogeneity in V_O show that men and those at a greater distance are less likely to fish Green Bay. The parameters are in Table D-4. The A-B parameters from this model are close to those from the homogeneous A-B model and main model, as is mean CV^G of $10.47, which is 7% higher than for the homogeneous full model.[17]

Because these models allow substitution in and out of Green Bay, mean E(CV^F) can also be compared across the models. For the first heterogeneous A-B-other specification, estimated mean E(CV^F) is $4.16, which is only one cent lower than $4.17 from the homogeneous full model. For the second heterogeneous specification, mean E(CV^F) is $4.13, which is 7% lower.

In theory, a random parameters specification for A-B parameters in the full model could be specified and estimated, although that is not done.[18] Because of the complexity and form of the likelihood function for the full model, simulation rather than quadrature as a means of estimation

16. For example, consider a male angler and a female angler who each live 5 miles from Green Bay. A man is willing to pay $8.74 per Green Bay fishing day for removal of FCAs and $3.62 for a doubling of the perch catch rate. A woman is willing to pay $12.14 per Green Bay fishing day for the removal of FCAs, but only $0.49 for a doubling of the perch catch rate.

17. As with the A-B model with classic heterogeneity, the estimated mean is a weighted average using the proportion of sample days as weights.

18. In contrast, a random term in V_O adds nothing, because $U_{Oi} = V_o + u_i + e_i$ is equivalent to $U_{Oi} = V_o + h_i$.

Table D-4
Parameter Estimates from Heterogeneous Nonrandom Models that Allow Substitution out of Green Bay

Parameter\Model	Heterogeneity in A-B[a]	Heterogeneity in V_O[b]
	Estimated Parameters (asymptotic *t*-statistics)	
Homogeneous parameters		
β_y	0.0446 (10.022)	0.0521 (19.313)
β_p	-0.0545 (-10.527)	-0.5345 (-13.150)
β_t	0.0206 (6.035)	-0.0244 (-9.296)
β_w	0.0386 (5.997)	-0.0294 (-10.294)
β_b	0.0050 (1.227)	-0.0255 (-8.297)
β_{FCA2}	-0.0481 (-14.277)	-0.0846 (-3.425)
β_{FCA3}	-0.2709 (-80.475)	-0.2508 (-5.843)
β_{FCA4}	-0.5409 (-160.440)	-0.5448 (-14.170)
β_{FCA5}	-0.6005 (-177.310)	-0.5853 (-18.225)
β_{FCA6}	-0.5369 (-159.446)	-0.5182 (-12.387)
β_{FCA7}	-0.7633 (-225.195)	-0.7453 (-25.813)
β_{FCA8}	-1.0245 (-300.755)	-1.0403 (-28.068)
β_{FCA9}	-1.2345 (-365.401)	-1.1384 (-24.772)
β_0	-0.7483 (-221.829)	-2.2961 (-24.500)
σ_0 or $\sigma_{0-\epsilon}$	3.199 (690.739)[c]	5.2441 (34.206)[c]
σ_{0-G}	NA[a]	4.1280 (25.304)[c]
Heterogeneous parameters		
β_{pg}	-0.3721 (-107.489)	
β_{tg}	-0.0322 (-5.042)	
β_{wg}	-0.0539 (-14.550)	
β_{bg}	-0.0187 (-5.844)	
β_{FCAg}	-0.2806 (-83.105)	
β_{pd}	-8.067e-4(-39.345)	
β_{td}	-9.212e-5 (-4.946)	
β_{wd}	-1.958e-4 (-7.864)	
β_{bd}	-6.061e-5 (-4.299)	
β_{FCAd}	3.842e-4 (16.472)	
γ_1		1.0450 (13.669)
γ_2		4.581e-3 (16.572)

a. These results are for an *A-B-other* model that allows substitution out of Green Bay, but the RP data on the actual number of days at Green Bay is not included. Therefore, σ_{0-G} is not a parameter in this model. In addition, ϵ_0 and ϵ_{ij} are assumed to be uncorrelated, so σ_0 rather than $\sigma_{0-\epsilon}$ is estimated. See text for discussion.
b. These results are for a full model that does include RP data on the actual number of Green Bay days.
c. σ parameters were exponentiated in estimation to restrict them to be positive. *t*-statistics apply to the logged parameter estimates.

would be necessary. Estimating a full model with random parameters seems unnecessary because: 1) mean consumer surplus is robust across the different nonrandom specifications; and 2) the higher consumer surplus values from the random A-B models suggest that damages estimated by the nonrandom full model are conservative.

D.3 DETAILS ON HERMITE POLYNOMIAL QUADRATURE

Hermite polynomial quadrature is a method of approximating integrals of functions on $(-\infty,\infty)$ with integrands that take the form presented in Equation 12. It is based on standard Gaussian methods. Consider first only one random parameter, in which case the approximation to Equation 12 is:

$$\int_{-\infty}^{\infty} e^{-n^2} g(n)dn = \sum_{m=1}^{M} w_m g(n_m) + R_m. \tag{16}$$

Here, ν_m is the mth zero of the Hermite polynomial $H_m(\nu)$, m is the number of evaluation points, and ω_m is the mth weight, given by:

$$w_m = \frac{2^{m-1} m! \sqrt{p}}{m^2 [H_{m-1}(n_m)]^2}. \tag{17}$$

The remainder is:

$$R_m = \frac{m! \sqrt{p}}{2^m (2m)!} g^{(2m)}(x), 0 < x < \infty. \tag{18}$$

Abramowitz and Stegun (1964) present ν_m and ω_m for various m in tabular form.

Let $\Delta x_{ij} = x_{ij}^{3-k_{ij}} - x_{ij}^{k_{ij}}$ and indicate the elements of this vector with superscripts. That is, Δx_{ij}^k is the kth element of Δx_{ij}^k. Suppose without loss of generality that the single varying parameter is the first. Then $g(\nu)$ is:

$$g(n) = p^{-1/2} \prod_{j=1}^{J} \Phi\left[(2s_e^2)^{-1/2}(-b'\Delta x_{ij} - \sqrt{2} s_u n \Delta x_{ij}^1)\right] \tag{19}$$

Notice the necessary change of variable to accommodate the fact that the normal kernal is $e^{-\frac{u^2}{2s_u^2}}$ and not e^{-u^2}.

For the case of two (or more) varying parameters the elements in the random vector ν_i in Equations 9 and 10 are treated separately (and denoted here by subscript), and the numerical integration is done from the inside out. Without loss of generality, suppose the two varying parameters are the first and the second. Then Equation 11 becomes:

$$P_i = \int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty}\prod_{j=1}^{J} F\,[(2s_e)^{-1/2}(-b\,'Dx_{ij} - \sqrt{2}s_{u_1}n_{1i}Dx_{ij}^{1} - \sqrt{2}s_{u_2}n_{2i}Dx_{ij}^{2})]\exp(-n_{2i}^{2})dn_{2i}\right]\bullet \exp(-n_{1i}^{2})dn_{1i}, \tag{20}$$

where s_{u_k} is the standard deviation of random parameter k. The integral inside the brackets is similar to the single varying parameter case, and can be evaluated in that manner. Call this quantity $h(\nu_1)$. It is a function of β, s_{u_1}, s_{u_2}, and ν_1, but not a function of ν_2 (recall σ_ϵ is not identified in this model). Equation 9 may be written:

$$P_i = \int_{-\infty}^{\infty} h(n_{1i})\exp(-n_{1i}^{2})dn_{1i} \tag{21}$$

which again can be evaluated as a single quadrature. The number of function evaluations increases exponentially. That is, if five function evaluations are used when there is a single varying parameter, then 25 are used for two, 125 for three, etc. The approximation of the double integral is:

$$P_i \approx p^{-1}\sum_{m_1=1}^{M_1} w_{m_1}\left[\sum_{m_2=1}^{M_2} w_{m_2}\, g(n_{2m_2}, n_{1m_1})\right] \tag{22}$$