JOURNAL OF ENVIRONMENTAL ECONOMICS AND MANAGEMENT 26, 257–270 (1994)

What Is Consumer's Surplus *Per Day of Use*, When Is It a Constant Independent of the Number of Days of Use, and What Does It Tell Us about Consumer's Surplus?

EDWARD R. MOREY[1]

Department of Economics, Campus Box 256, University of Colorado, Boulder, Colorado 80309-0256

Received June 13, 1991; revised May 14, 1993

An individual's consumer's surplus *per day of use* for a change in the price of recreational site is the price change, so it is a constant, independent of the number of days of use. Consumer's surplus *per day of use* for a change in a site's characteristics is not, in general, a constant. When a constant compensating variation per day of use exists, it multiplied by the number of days at the site in the original state (proposed state) bounds the compensating variation, CV, from below (above). The average of these two approximations is an *almost* second-order approximation to the CV. Simulations indicate the approximation biases can be large.

Consumer's surplus *per day of use* is a common way to express the benefits a representative individual derives from a recreational site. The U.S. Forest Service uses consumer's surplus per day of use for the existence of a site as the basic measure of that site's recreational value. Walsh *et al.* [15] survey 20 years of empirical research on the recreational value of our National Forests. They note, "The standard unit of measurement is an activity day, defined as one person on-site for any part of a calendar day" (p. 176). Derivation of per day of use measures is common in both the travel-cost and contingent valuation literature and is particularly common in the discrete-choice variants of these methodologies. A few examples are Bockstael *et al.* [3], Carson *et al.* [6], Cameron [4], Cameron and James [5], and Morey *et al.* [13].

Why the attraction to consumer's surplus per day of use when the desired welfare measure for policy analysis is not consumer's surplus per unit consumed, but instead consumer's surplus over some time interval? For a given time period, such as a year, the policymaker wants to know how each individual values a change in a site's price or characteristics rather than his or her value per day of use for that change.[2] However, policymakers and economists alike are attracted to per day of use measures for a number of reasons, one being consumer's surplus per day of use lends itself to use in *benefit transfers*. The notion is that once a representative individual's consumer's surplus per day of use has been estimated for Xing at one

[1]I thank Tayler Bingham, Neil Bruce, Erwin Diewert, Robert Rowe, V. Kerry Smith, and the participants of the AERE conference on benefits transfers (Snowbird, Utah, June 1992). I also thank the two anonymous referees. These individuals forced me to vigorously defend the arguments in this paper.

[2]Consumer's surplus is defined here as either the compensating variation or the equivalent variation associated with the change. These are always defined over some time period such as a year or season.

0095-0696/94 $6.00

site, where X is a recreational activity such as fishing or hiking, the analyst can obtain that individual's consumer's surplus for the site, or any similar site, by multiplying consumer's surplus per day of use at the first site by the number of days spent Xing at the site to be valued.

The purpose of this paper is to delve into the concept of consumer's surplus *per unit of use* and to determine its relationship to consumer's surplus. Does consumer's surplus per unit of use stand alone as a well-defined concept, and if so, should it be the standard-bearer for transferring benefit measures from one site to another?

I investigate consumer's surplus per day of use for a change in a site's price, and consumer's surplus per day of use for a change in a site's characteristics.[3] The theoretical results presented are general in that they hold for all demand models that explain days or trips to a recreational site. First, I demonstrate that consumer's surplus per day of use for a change in the price of a recreational site is the price change, so it is a constant, independent of the number of days the site is visited. Later, I demonstrate that consumer's surplus per day of use for a change in a site's characteristics is only a constant independent of the number of days at the site if one adopts the restrictive assumption that the utility an individual receives from a day at the site is independent of the number of days at the site. This restrictive assumption is the norm in discrete-choice models of recreational demand, but completely alien to standard neoclassical models of recreational demand that assume continuous choice and a diminishing marginal rate of substitution between trips to the site and all other commodities. A consumer's surplus per day of use that varies as a function of the number of days at a site is not terribly useful for approximating the consumer's surplus associated with a change in a site's characteristics.

When a constant compensating variation per day of use exists, it multiplied by the predicted number of days in the original state (proposed state) is a Laspeyres index (Paasche index) that bounds the compensating variation, CV, from below (above). The first approximation is a linear approximation to the CV and the second approximation is a linear approximation to the equivalent variation, EV. The average of these two approximations is an *almost* second-order approximation to the CV and is akin to the famous Harberger triangle. Simulation results indicate the bias in the linear approximations can be small or large, and the bias in the average of these two linear approximations, while often quite small, can be large if the proposed change will result in a large percentage change in the predicted number of days at the site.

1. THE CV PER DAY OF USE FOR A PRICE CHANGE

The purpose of this section is two-fold. First, to demonstrate that the CV per day of use for a price change is that price change, so it is a constant independent of the number of days of use. Second, to demonstrate that when CV per day of use

[3]Consumer's surplus per day of use for the availability of a site can be interpreted as the consumer's surplus per day of use for an increase in a site's price from its current level to the minimum price that would make demand zero.

is a constant, it can be used, along with information on the number of days at the site in the initial and proposed states, to *approximate* the CV for that change.[4]

Begin our examination of these issues with a thought experiment. Consider the maximum you would pay to have the price you pay for the next Coke you drink reduced by 50 cents. Your answer is 50 cents. Further note that this is how much you would pay each and every time you purchase a Coke to have the price of that Coke reduced by 50 cents. Fifty cents is your consumer's surplus *per unit of use* for having the price of Coke reduced by 50 cents (i.e., it's your *per Coke* consumer's surplus for the price reduction). It is a constant, independent of the number of Cokes you drink.

Consider now a similar thought experiment for a reduction in the cost of a day at a recreational site. For simplicity, assume a world of three commodities: two types of activities, days at a recreational site and days at home, and a numeraire good that can be consumed anywhere. What is the maximum amount an individual would pay each time he or she spent a day at the site to have the cost of that day reduced from P_d^0 to P_d^1, where P_d is the cost of a day at the site? The answer is $(P_d^0 - P_d^1)$, which is the individual's constant per day of use compensating variation for the price change, denoted $\mathrm{CVPD_U}$.[5]

Graphically, $\mathrm{CVPD_U}$ is represented in Fig. 1 as the vertical distance *ab*, whereas the individual's compensating variation, CV, associated with the change is the area $P_d^0acP_d^1$. Obviously, $\mathrm{CVPD_U} \neq \mathrm{CV}$. The issue is therefore how the CV can be derived, *or approximated*, from the $\mathrm{CVPD_U}$. Consider multiplying $\mathrm{CVPD_U}$ by the number of days at the site.[6] What is obtained depends on whether $\mathrm{CVPD_U}$ is multiplied by the number of days at the site in the original state, the number of days in the proposed state, or some average of the two. Define $\mathrm{CV_U^0} \equiv (D_U^0 \times \mathrm{CVPD_U})$, where D_U^0 is the number of days at the site when $P_d = P_d^0$. Graphically, $\mathrm{CV_U^0}$ is the area $P_d^0abP_d^1$. Define $\mathrm{CV_U^1} \equiv (D_U^1 \times \mathrm{CVPD_U})$, where D_U^1 is the predicted number of days when $P_d = P_d^1$. Graphically, $\mathrm{CV_U^1}$ is the area $P_d^0deP_d^1$. As Fig. 1 suggests, $\mathrm{CV_U^0}$ bounds the CV from below, and $\mathrm{CV_U^1}$ bounds the CV from above. This is true for both price increases and price decreases. Specifically, if $(P_d^0 - P_d^1) > 0$, $\mathrm{CV_U^1} \geq \mathrm{CV} \geq \mathrm{CV_U^0} \geq 0$, and if $(P_d^0 - P_d^1) < 0$, $0 \geq \mathrm{CV_U^1} \geq \mathrm{CV} \geq \mathrm{CV_U^0}$. Therefore, for price increases, $\mathrm{CV_U^0}$ ($\mathrm{CV_U^1}$) will overestimate (underestimate) the CV in absolute terms.

[4]Section 2 identifies conditions under which the CV per day for a change in a site's characteristics is a constant.

[5]One could loosely refer to $\mathrm{CVPD_U}$ as the *per trip* CV, a reference that would be exact if all trips were one day. Define CV *per day*, CVPD, as CV divided by the number of days in the time period (e.g., year) and note that $\mathrm{CVPD} \neq \mathrm{CVPD_U}$.

[6]Considering per day of use consumer's surplus measures, Bockstael *et al.* [3] state, "The calculation of CV according to equation (20) yields an estimate of the compensating variation *per choice occasion* for the household. To obtain annual or seasonal benefit estimates this number must be multiplied by the number of trips the individual takes" (pp. 10–28). In the same vein, Carson *et al.* [6] state, "The benefit is measured in terms of the maximum amount of money the individual would be willing to pay to ensure that the alternative is available whenever he makes a fishing choice. We therefore obtain an estimate of benefit per choice occasion, i.e., per fishing trip to any site, not just per-trip to the particular site of interest. Because our resident angler model is estimated on a weekly basis, the benefit to an individual is the benefit per choice occasion during that week, multiplied by the predicted number of trips (choice occasions) that week" (pp. 8–23).

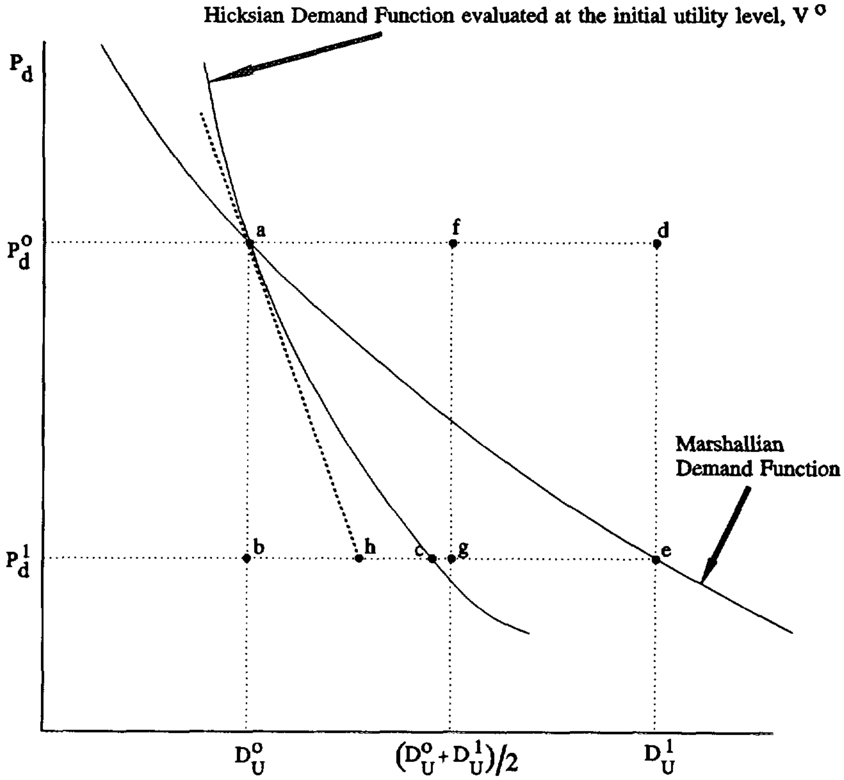

FIG. 1. D_U is the number of days at the site.

THEOREM 1.

$$CV_U^0 \equiv D_U^0(P_d^0 - P_d^1) \le CV \tag{1}$$

$$CV_U^1 \equiv D_U^1(P_d^0 - P_d^1) \ge CV. \tag{2}$$

Proof that $D_U^0(P_d^0 - P_d^1) \le$ CV. Define the indirect utility function for the season as $V = V(Y, P_h, P_d)$, where Y is income, P_h is the cost of each day at home, $V^0 \equiv V(Y^0, P_h^0, P_d^0)$, and $V^1 \equiv V(Y^0, P_h^0, P_d^1)$. The indirect utility function identifies maximum utility as a function of income and prices. Dual to this indirect utility function is the expenditure function $E = E(V, P_h, P_d)$; it identifies the minimum expenditures required to achieve a utility level of V given P_h and P_d.[7] Define D_U as the number of days at the site, H as the number of days at home, and let N denote the quantity of the numeraire consumed; i.e., $N \equiv Y - P_d D_U - P_h H$.

By definition, the CV for a change from $\{P_h^0, P_d^0\}$ to $\{P_h^0, P_d^1\}$ is

$$CV = E(V^0, P_h^0, P_d^0) - E(V^0, P_h^0, P_d^1). \tag{3}$$

[7]For more details on indirect utility functions, expenditure functions, and the CV and EV in terms of these functions, see, for example, Varian [14].

By definition of the expenditure function

$$E(V^0, P_h^0, P_d^0) = P_d^0 D_U^0 + P_h^0 H^0 + N^0 \quad (4)$$

$$E(V^1, P_h^0, P_d^1) = P_d^1 D_U^1 + P_h^0 H^1 + N^1. \quad (5)$$

Substitute Eq. (4) into Eq. (3) to obtain

$$\mathrm{CV} = \left(P_d^0 D_U^0 + P_h^0 H^0 + N^0\right) - E\left(V^0, P_h^0, P_d^1\right). \quad (6)$$

Now note that

$$E\left(V^0, P_h^0, P_d^1\right) \leq P_d^1 D_U^0 + P_h^0 H^0 + N^0 \quad (7)$$

because D_U^0, H^0, and N^0 are by definition capable of producing V^0. Therefore, $P_d^1 D_U^0 + P_h^0 H^0 + N^0$ are sufficient expenditures to produce V^0 given P_d^1 and P_h^0. However, $E(V^0, P_h^0, P_d^1)$ is by definition the minimum expenditures required to produce V^0 given P_d^1 and P_h^0.

Replacing $E(V^0, P_h^0, P_d^1)$ in Eq. (6) with the right-hand side of Eq. (7), and noting the inequality in Eq. (7), one obtains

$$\mathrm{CV} \geq \left(P_d^0 D_U^0 + P_h^0 H^0 + N^0\right) - \left(P_d^1 D_U^0 + P_h^0 H^0 + N^0\right) = D_U^0\left(P_t^0 - P_t^1\right). \quad \text{Q.E.D.} \quad (8)$$

The proof that $\mathrm{CV}_U^1 \equiv D_U^1(P_d^0 - P_d^1) \geq \mathrm{CV}$ is analogous to the proof that $\mathrm{CV}_U^0 \equiv D_U^0(P_d^0 - P_d^1) \leq \mathrm{CV}$. CV_U^0 can be viewed as a Laspeyres quantity approximation to the CV because it evaluates the impact of the price change restrictively assuming the number of days at the site will remain at their initial level, D_U^0. In contrast, CV_U^1 can be viewed as a Paasche quantity approximation to the CV because it evaluates the impact of the price change restrictively assuming the individual spends D_U^1 days at the site both before and after the price change.

An immediate corollary to Theorem 1 is that CVPD_U is not the CV divided by the number of days of use in either the initial state or the proposed state. If it were, Theorem 1 would be contradicted. Therefore, it is misleading to view CVPD_U as an average of the CV in terms of days of use.[8]

Note that from Eq. (6), Eq. (7), and the definition of CV_U^0, it follows that

$$\mathrm{CV} - \mathrm{CV}_U^0 = P_d^1 D_U^0 + P_h^0 H^0 + N^0 - E\left(V^0, P_h^0, P_d^1\right) \geq 0. \quad (9)$$

As Eq. (9) indicates, the bias in CV_U^0 is how much expenditures to produce V^0

[8]To clarify, for any change in the price and or characteristic of a site, there will be a CV. If this CV is known, one can divide it by X, to get average CV in terms of units of X. X could, for example, be the number of days at the site in the initial state, the number of days in the proposed state, or the number of elephants in Africa. However, CVPD_U does not equal CV/(number of days at the site in the initial state) or CV/(number of days at the site in the proposed state). For example, in Fig. 1, CVPD_U, the vertical distance ab, is less than CV/(number of days at the site in the initial state), the (area $P_d^0 ac P_d^1)/D_U^0$. Further note that if one knows X and the average CV in terms of units of X, one can multiply them to get the CV itself, not an approximation. However, if the CV is known, there is no point in first dividing and then multiplying it by X.

would change at the proposed prices if the individual is allowed to adjust his allocation from $\{D_U^0, H^0, N^0\}$ to $\{D_U^1, H^1, N^1\}$.

CV_U^0 is a linear approximation to the CV for a change in P_d (Boadway and Bruce [2]), that is

$$\begin{aligned} CV &= E(V^1, P_h^0, P_d^1) - E(V^0, P_h^0, P_d^1) \\ &\approx E(V^1, P_h^0, P_d^1) - \left[E(V^0, P_h^0, P_d^0) + \frac{\partial E(V^0, P_h^0, P_d^0)}{\partial P_d}(P_d^1 - P_d^0) \right] \quad (10) \end{aligned}$$

by Taylor's Theorem

$$= -\frac{\partial E(V^0, P_h^0, P_d^0)}{\partial P_d}(P_d^1 - P_d^0)$$

$$\text{since } E(V^1, P_h^0, P_d^1) = E(V^0, P_h^0, P_d^0) = Y^0$$

$$= D_U^0(P_d^0 - P_d^1) \equiv CV_U^0$$

by Shepard's Lemma.[9] By an analogous argument, CV_U^1 is a linear approximation to the equivalent variation.

Note that CV_U^0 is akin to another linear approximation to the CV, denoted CV_{Hk}. CV_{Hk} is essentially due to Hicks [11, 12]. For the change from P_d^0 to P_d^1, this Hicksian approximation to the CV, which is in terms of quantity changes rather than price changes, is (Diewert [8])[10]

$$CV \approx CV_{Hk} = P_d^1(D_U^1 - D_U^0) + P_h^0(H^1 - H^0) + (N^1 - N^0). \quad (11)$$

Therefore CV_U^0 might be labeled a Hicksian *price-change* approximation to the CV. My perception is that economists interested in the valuation of recreational sites prefer price-change approximations to the CV over quantity-change approximations. This is because it is easier to observe the initial demand for a site than to observe, or predict, the changes in the quantity demanded for the site and all other commodities that would result from a change in the site's price.[11]

Summarizing to here, $D_U^0(P_d^0 - P_d^1)$ and $D_U^1(P_d^0 - P_d^1)$ are respectively lower and upper bounds on the CV for this price change, and $D_U^0(P_d^0 - P_d^1)$ is, in addition, a linear approximation to the CV for this price change. These results make consumer's surplus per day of use for a price change, $(P_d^0 - P_d^1)$, useful.

Unfortunately, neither CV_U^0 or CV_U^1 will always closely approximate the CV. Put simply, the actual degree of bias in these linear approximations depends on the individual's preferences and the magnitude of the price change. The bias can be

[9]By Shepard's Lemma, the Hicksian demand for days at the site given V^0, P_h^0, and P_d^0 is $\partial E(V^0, P_h^0, P_d^0)/\partial P_d$. But since $V^0 \equiv V(Y^0, P_h^0, P_d^0)$, the Hicksian demand at $\{Y^0, P_h^0, P_d^0\}$ is D_U^0, the Marshallian demand at $\{Y^0, P_h^0, P_d^0\}$.

[10]Note that CV_{Hk} cannot be identified as an area in Fig. 1 because it is expressed in terms of changes in H and N.

[11]For more details see footnote 14.

small or large. For example, in Fig. 1 the bias is visually significant.[12] Intuitively, the bias in CV_U^0 and CV_U^1 results because neither measure considers the substitutability between days at home and days at the site. The degree of bias in each of these measures is an increasing function of the *marginal rate of substitution* between days at home and days at the site and of the magnitude of the price change; the greater the change in D_U that will result from the proposed price change, the greater the bias.

In contrast to these linear approximations, the average of CV_U^0 and CV_U^1 is *almost* a second-order approximation to the CV for a change in P_d. Denote this average CV_U^{ave} as

$$\begin{aligned} CV_U^{ave} &= \tfrac{1}{2}(CV_U^0 + CV_U^1) = (P_d^0 - P_d^1)\tfrac{1}{2}(D_U^1 + D_U^0) \\ &= D_U^0(P_d^0 - P_d^1) + \tfrac{1}{2}(P_d^0 - P_d^1)(D_U^1 - D_U^0). \end{aligned} \tag{12}$$

CV_U^{ave} will usually, but not always, better approximate the CV than either CV_U^0 or CV_U^1.[13] In terms of Fig. 1, CV_U^{ave} is the area $P_d^0 fg P_d^1$.

In contrast to CV_U^{ave}, an *exact* second-order approximation to the CV for a change in P_d (Boadway and Bruce [2]) is

$$\begin{aligned} CV &= E(V^1, P_h^0, P_d^1) - E(V^0, P_h^0, P_d^1) \\ &\approx E(V^1, P_h^0, P_d^1) - \left[E(V^0, P_h^0, P_d^0) + \frac{\partial E(V^0, P_h^0, P_d^0)}{\partial P_d}(P_d^1 - P_d^0)\right. \\ &\qquad \left. + \frac{1}{2}\frac{\partial^2 E(V^0, P_h^0, P_d^0)}{\partial P_d^2}(P_d^1 - P_d^0)^2\right] \qquad \text{(by Taylor's Theorem)} \\ &= -\frac{\partial E(V^0, P_h^0, P_d^0)}{\partial P_d}(P_d^1 - P_d^0) - \frac{1}{2}\frac{\partial^2 E(V^0, P_h^0, P_d^0)}{\partial P_d^2}(P_d^1 - P_d^0)^2 \end{aligned}$$

[12] CV_U^0 is less than the CV by the area *acb* and CV_U^1 is greater than the CV by the area *adec*.

[13] Also consider the geometric and harmonic means of CV_U^0 and CV_U^1; CV_U^{ave} is the arithmetic mean. The geometric mean of CV_U^0 and CV_U^1 is

$$CV_U^{G-ave} = \left[(CV_U^0)(CV_U^1)\right]^{1/2},$$

and the harmonic mean is

$$CV_U^{H-ave} = \left[\left(\frac{1}{2}\right)\left(\frac{1}{CV_U^0}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{CV_U^1}\right)\right]^{-1}.$$

It is the case that Abramowitz and Stegun [1]

$$|CV_U^{H-ave}| \le |CV_U^{G-ave}| \le |CV_U^{ave}|.$$

The arithmetic, geometric, and harmonic means are often close in value. However, there are exceptions, as becomes obvious at the end of Section 1 where I report simulation results. In general, if these three means differ significantly, none of them are good approximations to the CV.

since

$$E(V^1, P_h^0, P_d^1) = E(V^0, P_h^0, P_d^0) = Y^0$$
$$= D_U^0(P_d^0 - P_d^1) + \frac{1}{2}\frac{\partial D_U(V^0, P_h^0, P_d^0)}{\partial P_d}(P_d^1 - P_d^0)(P_d^0 - P_d^1) \quad (13)$$

by Shepard's lemma. Comparing Eqs. (12) and (13), the difference between CV_U^{ave} and this exact second-order approximation to the CV results because $(D_U^1 - D_U^0)$ approximates, but does not equal, $(P_d^1 - P_d^0)$ multiplied by the slope of the Hicksian demand function for D_U evaluated at the initial utility level and prices. In Fig. 1, this exact second-order approximation to the CV is the area $P_d^0ahP_d^1$. It's a better approximation than CV_U^{ave} but requires more information to calculate; calculation requires the slope of the Hicksian demand function for D_U evaluated at the initial utility level and prices.

Note that a different *almost* second-order approximation to the CV is the well-known Harberger triangle. For the single price change P_d^0 to P_d^1, the Harberger triangle is $\frac{1}{2}(P_d^0 - P_d^1)(D_U^1 - D_U^0) + P_h^0(H^1 - H^0) + (N^1 - N^0)$.[14] The difference between CV_U^{ave} and the Harberger triangle is that CV_U^{ave} is an almost second-order approximation to the CV in terms of the price change, and the Harberger triangle is an almost second-order approximation to the CV in terms of the quantity changes. In this sense CV_U^{ave} might be labeled the price-change equivalent to the Harberger triangle.

Summarizing the last few paragraphs, a constant CV per day of use can be used to obtain an almost second-order approximation to the CV by multiplying the constant CV per day of use by the average number of days at the site in the initial and proposed states. In general, this approximation is better than the approximation obtained by multiplying CV per day of use by the number of days at the site in one of the states.

[14] Note that the Harberger triangle, like CV_{Hk}, cannot be identified as an area in Fig. 1 because it is expressed in terms of changes in H and N.

For more details on the properties of the Harberger triangle see Harberger [10], Diewert [7–9], and Weitzman [16].

Both the Harberger triangle and the simpler Hicksian approximation CV_{Hk} (Eq. (11)) belong to the class of approximations that require observations on (or estimates of) p^0, p^1, x^0, and x^1, where, in our notation, $p \equiv \{p_d, p_n, 1\}$ and $x \equiv \{D_U, H, N\}$. For example, when only the price of a day at the site changes, P_d^0 to P_d^1, the calculation of the Harberger triangle requires P_d^0, P_h^0, D_U^0, H^0, N^0, P_d^1, D_U^1, H^1, and N^1.

In contrast, calculation of CV_U^0 requires just P_d^0, P_d^1, and D_U^0, CV_U^1 requires just P_d^0, P_d^1, and D_U^1, and CV_U^{ave} requires just P_d^0, P_d^1, D_U^0, and D_U^1.

Enhancements to the Harberger triangle approximation have recently come from both Weitzman [16] and Diewert [9]. Extensions of the Harberger triangle involve finding functional forms for the expenditure function which have the property that the EV is exactly equal to a known function of p^0, p^1, x^0, and x^1 (Diewert [9]), in which case, the EV function is exact, rather than an approximation, for some known preference ordering. If, in addition, that known preference ordering is represented by an expenditure function that is *flexible*, the EV function is said to be a *superlative* welfare indicator.

Three superlative welfare indicators are the *Fisher welfare change indicator* $W_F(p^0, p^1, x^0, x^1)$ and the *normalized quadratic indicators* $W_0(p^0, p^1, x^0, x^1)$ and $W_1(p^0, p^1, x^0, x^1)$. The Fisher welfare indicator is exact for a *transformed quadratic expenditure function* (Diewert [9, Eqs. (39) and (43)], and the normalized quadratic indicators are exact for two particular parameterizations of the *normalized quadratic expenditure function* (Diewert [9, Eqs. (48)–(53)].

To get a feel for how large the biases in CV_U^0, CV_U^1, and CV_U^{ave} can be, I ran many simulations. Simulations tell us nothing about how small or large the bias will be in any particular real world example. They are by definition assumption specific; a particular preference ordering is assumed, and then the bias is determined for different price changes for that preference ordering. The simulations reported here are based on a simple repeated discrete-choice random-utility model of the probability of visiting the site on any given day. Simulations I did using a continuous-choice model exhibit similar degrees of bias. Figure 1 is one example of the degrees of bias that can result in a continuous-choice model. I chose to report numerically the simulation results from the discrete-choice model, rather than a continuous-choice model, because the discrete-choice simulations can also be used to make an important point about when the CV per day of use for a change in a site's characteristics is a constant. No claim is made that the discrete-choice model reflects truth.

One hundred price changes, P_d^0 to P_d^1, were simulated. For each, I calculated CV, CV_U^0, CV_U^1, CV_U^{ave}, $Prob_U^0$ and $Prob_U^1$, where $Prob_U$ is the per day probability of visiting the site. The outcomes for five representative price changes are reported in Table I. They are referenced in the text by case number. The largest bias I generated resulted from a price reduction that causes the per day probability of visiting the site to increase from effectively zero to 2%. For this case (case 1), CV = \$18.25, CV_U^0 = \$0.0025, and CV_U^1 = \$203.40. For comparison, a price reduction that causes the probability to increase from 4 to 9% (case 2) generated a CV of \$58.95, a CV_U^0 of \$35.34, and a CV_U^1 of \$90.56, and a price increase that causes

For the change from P_d^0 to P_d^1, $W_F(p^0, p^1, x^0, x^1)$ [9, Eq. (45)] simplifies to

$$W_F = \left[\left[\frac{(P_d^1 D_U^1 + P_h^0 H^1 + N^1)(P_d^0 D_U^1 + P_h^0 H^1 + N^1)}{(P_d^1 D_U^0 + P_h^0 H^0 + N^0)(P_d^0 D_U^0 + P_h^0 H^0 + N^0)}\right]^{1/2} - 1\right](P_d^0 D_U^0 + P_h^0 H^0 + N^0),$$

$W_0(p^0, p^1, x^0, x^1)$ [9, Eq. (56)] simplifies to

$$W_0 = \left[\left(\frac{1}{2}\right)\frac{(P_d^1 D_U^1 + P_h^0 H^1 + N^1)}{(P_d^1 D_U^0 + P_h^0 H^0 + N^0)} + \left(\frac{1}{2}\right)\frac{(P_d^0 D_U^1 + P_h^0 H^1 + N^1)}{(P_d^0 D_U^0 + P_h^0 H^0 + N^0)} - 1\right](P_d^0 D_U^0 + P_h^0 H^0 + N^0),$$

and $W_1(p^0, p^1, x^0, x^1)$ [9, Eq. (57)] simplifies to

$$W_1 = \left[\left[\left(\frac{1}{2}\right)\frac{(P_d^1 D_U^0 + P_h^0 H^0 + N^0)}{(P_d^1 D_U^1 + P_h^0 H^1 + N^1)} + \left(\frac{1}{2}\right)\frac{(P_d^0 D_U^0 + P_h^0 H^0 + N^0)}{(P_d^0 D_U^1 + P_h^0 H^1 + N^1)}\right]^{-1} - 1\right](P_d^0 D_U^0 + P_h^0 H^0 + N^0).$$

If, in addition to observations on (or estimates of) P_d^0, P_d^1, D_U^0, and D_U^1, one also has observations on (or estimates of) P_h^0, H^0, N^0, H^1, and N^1, one should consider using this additional information and approximate the welfare change with either the Fisher welfare change indicator or one of the normalized quadratic indicators. They will likely provide better approximations to the CV than CV_U^{ave}. It should be fairly easy to calculate these estimates if one assumes $P_h = 0$, in which case all of the terms multiplicative in P_h drop out and $N = Y - P_d D_U$, where Y = income.

TABLE I
Five Representative Simulation Results

Case	CV	CV_U^0	CV_U^1	CV_U^{ave}	CV_U^{G-ave}	CV_U^{H-ave}	$Prob_U^0$	$Prob_U^1$
1	\$18.25	\$0.0025	\$203.40	\$101.70	\$0.713	\$0.0049	≈ 0%	2%
2	\$58.95	\$35.34	\$90.56	\$62.95	\$56.57	\$50.84	4%	9%
3	−\$22.59	−\$35.34	−\$13.30	−\$24.32	−\$21.67	−\$19.32	4%	1%
4	\$0.3233	\$0.3205	\$0.3262	\$0.3234	\$0.3233	\$0.3233	2%	2.04%
5	−\$101.30	−\$113.60	−\$89.50	−\$101.50	−\$100.83	−\$100.12	34%	26%

the per day probability to decrease from 4 to 1% (case 3) generates a CV of −\$22.59, a CV_U^0 of −\$35.34, and a CV_U^1 of −\$13.30. For the 100 simulations, neither CV_U^0 or CV_U^1 closely approximates the CV unless the price change caused the probability to change by less than 10%, and then CV, CV_U^0, and CV_U^1 are all effectively zero. A typical example is case 4 where the price decrease caused the per day probability to increase from 2 to only 2.04% (a 2% change). This price change generated a CV of \$0.3233, a CV_U^0 of \$0.3205, and a CV_U^1 of \$0.3262. Alternatively, a price increase that caused the per day probability to decrease from 34 to 26% (a 30% change) generated a CV of −\$101.30, a CV_U^0 of −\$113.60, and a CV_U^1 of −\$89.50 (case 5). These simulation results are just examples, but they do indicate the potential for a bias and a bias which increases as the significance of the price change increases.

CV_U^{ave} much more closely approximates the CV. For the five simulation results noted above, the CV's and their corresponding CV_U^{ave}'s are {\$18.25 and \$101.70}, {\$58.95 and \$62.95}, {−\$22.59 and −\$24.32}, {\$0.3233 and \$0.3234}, and {−\$101.30 and −\$101.50}.[15] Except for the first set, CV and CV_U^{ave} are all similar. In the first case, there is a five-fold difference between the CV and the CV_U^{ave}. Again, these simulation results should not be taken too seriously, but they do suggest that the CV_U^{ave} closely approximates the CV except in cases in which the price change will cause a many-fold change in the number of days at the site. However, this is a significant concern because any policy that increases demand from effectively zero to a small number of days will involve a large multiplicative change in total demand.

2. THE CV PER DAY OF USE FOR A CHANGE IN A SITE'S CHARACTERISTICS IS NOT, IN GENERAL, A CONSTANT

Up to this point we have only considered the CV associated with a change in the price of a day at the site. However, often we need to estimate or approximate the CV associated with a change in the characteristics of a site. In Section 1 I demonstrated that if a constant CV per day of use exists, it can be used, along with the number of days at the site either before or after the change, to approximate the CV. In Section 1 I also demonstrated that the CV per day of use for a price change is that price change, so is therefore a constant. The question addressed in

[15] The geometric mean CV_U^{G-ave} and the harmonic mean CV_U^{H-ave} for these cases are also reported in Table I. Except for case 1, they are both reasonable approximations to the CV. For more details see footnote 13.

this section is whether a constant CV per day of use will exist if the proposed change involves a change in the characteristics of the site. The answer is, in general, no.

To begin our investigation of this question, again consider our Coke-drinking thought experiment, but now determine how much you would pay to have the number of calories in the ith Coke that you drink reduced by 50%. Without loss of generality, I'll denote your answer α_i. This amount, α_i, is your CV for this calorie reduction *for the ith Coke consumed.* In general, this amount is not a constant. A constant *per Coke* CV, independent of the number of Cokes you choose to drink, only exists if $\alpha_i = \alpha$ for all i, in which case α is a constant CV *per Coke consumed.* In cases such as our first thought experiment where the change is solely a price change, $\alpha_i = \alpha =$ the price reduction for all i, but typically $\alpha_i \neq \alpha$ if the change involves a change in the characteristics of the commodity. For example, how much I would pay to have the calories reduced in the last Coke I drink will increase as I drink more Cokes.

By definition, $\alpha_i = \alpha$ only if how you value the change, in money terms, is independent of the number of Cokes you choose to drink. This must be true for a change in the price of a Coke, but what would make it true for a change in the characteristics of the Coke? Without loss of generality, assume a world of three commodities: two activities, drinking a Coke and not drinking a Coke, and a numeraire good, where both activities take all day. If you choose not to drink a Coke, you spend all of your income for the day on the numeraire. Otherwise, you allocate to the numeraire your income for the day minus the price of the Coke. If one restrictively assumes the utility you receive on a day is only a function of whether you drink a Coke that day, the amount of the numeraire consumed that day, and the characteristics of Coke, you will have a CV *per Coke consumed* for any change in the price and/or characteristics of Coke which is a constant independent of the number of Cokes you choose to drink. However, assuming the utility you receive from each Coke you drink is independent of the number of Cokes you drink is not terribly realistic. It is also inconsistent with any demand model that assumes a diminishing MRS between Cokes and all other commodities.

Now consider a similar thought experiment for a change in the characteristics of a recreational site. Again, without loss of generality, assume a world of three commodities: two types of activities, days at the recreational site and days at home, and a numeraire good that can be consumed anywhere. What is the maximum amount an individual would pay each and every time he or she spends a day at the site to have the characteristics of the site be C^1 rather than C^0? As our last thought experiment indicates, the individual will not in general be able to answer this question because there is no such amount. A constant CV *per day of use* for changes in the characteristics of the site, α, does not usually exist.

In a world with a recreational site but no Coke, $\alpha_i = \alpha$ only if how the individual values the change in the site in money terms is independent of the number of days he or she spends at the site. $\alpha_i = \alpha$ must be true for a change in the price of a day at the site but will not, in general, be true for changes in the characteristics of the site.

An individual's CV per day of use will be a constant in our world of three commodities *only if* one makes the additional assumption that the utility the individual receives on a day is only a function of whether he or she spends that day at the site, the amount of the numeraire consumed that day, and the characteristics of the site. In this case, $\alpha_i = \alpha$ for any change in the price of a day and/or change in the characteristics of the site. Note that when this assumption is made there is always a price change that would make the individual indifferent between that price change and the proposed change in the characteristics, and this price change is independent of the number of days spent at the site. One might denote this price change as the *quality-equivalent price change*. Therefore, when one adopts the restrictive assumption that the utility from a day at the site is independent of the number of days at the site, any change in the characteristics of a site can be *converted* into its *quality-equivalent price change*, and Theorem 1 and all the approximation results apply in terms of this quality-equivalent price change. The fact that a characteristics change can be converted into an equivalent price change when this restrictive assumption holds makes Theorem 1 and the approximation results particularly relevant to discrete-choice models of recreational demand. However, this restrictive assumption is inconsistent with all travel-cost models that assume a diminishing MRS between days at the site and other commodities.

When is it assumed that the utility from a day at the site is independent of the number of days at the site? The assumption that the utility the individual receives on a day is only a function of whether he or she spends that day at a site, the amount of the numeraire consumed that day, and the characteristics of the site is the basic assumption of many discrete-choice models of recreational demand. This is why a constant CV per day of use can be derived for changes in both prices and characteristics from most discrete-choice models of recreational demand.[16] Consider a simple dichotomous logit or probit model designed to predict the probability that an individual will visit a particular site on a given day. Such models are based on two conditional indirect utility functions. One specifies the utility received for the day if the site is visited, and the other specifies the utility received for the day if the site is not visited. From such a random-utility model one can derive a constant CV *per day* for any change in the price or characteristics of the site, denoted CVPD. This CV per day can, for example, be multiplied by the number of days in the year (365) to get the CV for the year. From this discrete-choice model one can also derive a constant CV per day of use, $CVPD_U$.[17] Note that CV per day of use is not the same thing as CV per day. CV per day of use is what CV per day would be if the individual *were constrained to spend the day at the site*, that is, the CV per day assuming the site will be visited. The individual is not constrained in this way.

Theorem 1 and the approximation results imply the following for the simple discrete-choice model of recreational demand outlined above: (1) the constant CV per day of use multiplied by the predicted number of days each year to the site in the original state is both a lower bound and a linear approximation to the CV associated with the change; (2) the constant CV per day of use multiplied by the

[16] For example, see the earlier references to Bockstael *et al.* [3], Carson *et al.* [6], and Morey *et al.* [13].

[17] For details on the derivation of CVPD and $CVPD_U$ from discrete-choice models see Morey *et al.* [13]. Most typically, $CVPD_U$ is derived from multiple-site discrete-choice models that model the choice of site given the decision to participate, but that do not model the participation decision. If, from such a model, one derives the consumer's surplus associated with a change in the characteristics of the site, that consumer's surplus is a per day of use measure because the individual is implicitly constrained to take a trip.

predicted number of days each year to the site in the proposed state is an upper bound on the yearly CV associated with the change, and a linear approximation to the yearly EV associated with the change; and (3) the constant CV per day of use multiplied by the average of the predicted number of days at the site in the two states is almost a second-order approximation to the CV associated with the change. The simulation results discussed earlier were all derived from a discrete-choice random utility model of this type. Therefore, even though my original discussion of simulation results described the CVs as those for price changes, they could for this model also be described as CVs resulting from changes in the characteristics of the site. This is true because any change in the characteristics of the site has a *quality-equivalent price change* if one assumes that the utility the individual receives on a day is only a function of whether he or she spends that day at a site, the amount of the numeraire consumed that day, and the characteristics of the site.

In contrast, CV per day of use for a policy that involves a change in the site's characteristics is not a constant if one adopts a travel-cost model that assumes a diminishing MRS between days at the site and other commodities, an assumption implicit in many travel-cost models. In these cases, Theorem 1 does not apply because it only applies to cases where CV per day of use is a constant. However, Theorem 1 still does apply to policies that just involve changes in the price of the site, because for price changes, CV per day of use is a constant in all travel-cost models.

CONCLUSIONS

Care is required when using consumer's surplus per day of use. A constant consumer's surplus per day of use exists for any change in the price of a day at a recreational site but does not, in general, exist if the change involves a change in the characteristics of the site. For characteristic changes, it is only a constant if the utility one gets from a day at the site is independent of the number of days at the site, a not terribly realistic assumption.

In addition, even when a constant consumer's surplus per day of use exists, multiplying it by the number of days at the site in the original state provides only a lower bound for the consumer's surplus, and multiplying it by the number of days at the site in the proposed state provides only an upper bound for the consumer's surplus. Simulations show the bias in these approximations can be small or large. The average of the two often closely approximates the consumer's surplus, but even this average can be significantly biased for many proposed policies.

The question of what can be done with a CV per day of use that is not a constant is left for future research.

REFERENCES

1. M. Abramowitz and I. A. Stegun (Eds.), "Pocketbook of Mathematical Functions, Abridged Edition of Handbook of Mathematical Functions," materials selected by M. Danos and J. Rafelski, Verlag Harri Deutsch, Thun (1984).
2. R. Boadway and N. Bruce, "Welfare Economics," Blackwell, Oxford (1984).
3. N. E. Bockstael, M. W. Hanemann, and I. E. Strand, Jr., "Measuring the Benefits of Water Quality Improvements Using Recreation Demand Models," Vol. II, prepared for the Office of Policy Analysis, U.S. Environmental Protection Agency, Washington, DC (1984).

4. T. A. Cameron, A new paradigm for valuing non-market goods using referendum data: Maximum likelihood estimation by censored logistic regression, *J. Environ. Econom. Management* **15**, 355–379 (1988).
5. T. A. Cameron and M. D. James, Efficient estimation methods for use with "close-ended" contingent valuation survey data, *Rev. Econom. Statist.* **69**, 269–276 (1987).
6. R. T. Carson, M. W. Hanemann, and T. Wegge, "Southcentral Alaska Sport Fishing Economic Study," report prepared by Jones and Stokes Associates, Sacramento, CA, for Alaska Department of Fish and Game, Anchorage, AK (1987).
7. W. E. Diewert, Harberger's welfare indicator and revealed preference theory, *Amer. Econom. Rev.* **66**(1), 143–152 (1976).
8. W. E. Diewert, Cost functions, *in* "The New Pelgrave Dictionary of Economics," (J. Eatwell, M. Milgate, and P. Newman, Eds.), Vol. I, Stockton Press, New York (1987).
9. W. E. Diewert, Exact and superlative welfare change indicators, *Econom. Inquiry* **30**, 562–582 (1992).
10. A. C. Harberger, Three basic postulates of applied welfare economics: An interpretive essay, *J. Econom. Lit.* **9**(3), 785–797 (1971).
11. J. R. Hicks, Consumer's surplus and index numbers, *Rev. Econom. Stud.* **9**, 126–137 (1942).
12. J. R. Hicks, "Value and Capital," 2nd ed., Clarendon, Oxford (1946).
13. E. R. Morey, R. D. Rowe, and M. Watson, A repeated nested-logit model of Atlantic salmon fishing, *Amer. J. Agr. Econom.* **75**, 578–592 (1993).
14. H. L. Varian, "Microeconomic Analysis," Third ed., Norton, New York (1992).
15. R. B. Walsh, D. Johnson, and J. Mckean, Review of outdoor recreation economic demand studies with non-market benefit estimates, 1968–1988, *in* "Advances in Applied Microeconomics," Vol. 5, "Recent Developments in Modeling of Technical Change; and Modeling Demand for and Valuation of Recreation Resources" (A. N. Link and V. K. Smith, Eds.), Jai Press, Greenwich, CT (1990).
16. M. L. Weitzman, Consumer's surplus as an exact approximation when prices are appropriately deflated, *Quart. J. Econom.* **103**(3), 543–553 (1988).